



Método eletromagnetométrico



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Métodos eletromagnéticos

- Resumo

Domínio do tempo

$$\vec{\nabla}^2 \vec{e} - \mu\epsilon \frac{\partial^2 \vec{e}}{\partial t^2} - \mu\sigma \frac{\partial \vec{e}}{\partial t} = 0$$

$$\vec{\nabla}^2 \vec{e} - \mu\sigma \frac{\partial \vec{e}}{\partial t} = 0$$

$$\vec{\nabla}^2 \vec{h} - \mu\epsilon \frac{\partial^2 \vec{h}}{\partial t^2} - \mu\sigma \frac{\partial \vec{h}}{\partial t} = 0$$

$$\vec{\nabla}^2 \vec{h} - \mu\sigma \frac{\partial \vec{h}}{\partial t} = 0$$

Domínio da frequência

$$\vec{\nabla}^2 \vec{E} + (\mu\epsilon\omega^2 - i\mu\sigma\omega)\vec{E} = 0$$

$$\vec{\nabla}^2 \vec{E} - i\mu\sigma\omega\vec{E} = 0$$

$$\vec{\nabla}^2 \vec{H} + (\mu\epsilon\omega^2 - i\mu\sigma\omega)\vec{H} = 0$$

$$\vec{\nabla}^2 \vec{H} - i\mu\sigma\omega\vec{H} = 0$$

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Métodos eletromagnéticos

Para 1D - Componente z

tempo

$$\vec{\nabla}^2 \vec{e} - \mu\sigma \frac{\partial \vec{e}}{\partial t} = 0$$

$$\frac{\partial^2 \vec{e}}{\partial z^2} - \mu\sigma \frac{\partial \vec{e}}{\partial t} = 0$$

$$\vec{\nabla}^2 \vec{h} - \mu\sigma \frac{\partial \vec{h}}{\partial t} = 0$$

$$\frac{\partial^2 \vec{h}}{\partial z^2} - \mu\sigma \frac{\partial \vec{h}}{\partial t} = 0$$

frequência

$$\vec{\nabla}^2 \vec{E} - i\mu\sigma\omega \vec{E} = 0$$

$$\frac{\partial^2 \vec{E}}{\partial z^2} - i\mu\sigma\omega \vec{E} = 0$$

$$\vec{\nabla}^2 \vec{H} - i\mu\sigma\omega \vec{H} = 0$$

$$\frac{\partial^2 \vec{H}}{\partial z^2} - i\mu\sigma\omega \vec{H} = 0$$

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Métodos eletromagnéticos

- Soluções da equação de onda - As equações do lado esquerdo são diferenciais de segunda ordem. Duas soluções básicas destas equações são de interesse.
- A primeira solução possui dependência senoidal com o tempo ($e^{i\omega t}$):

$$\frac{\partial^2 \vec{e}}{\partial z^2} - \mu\sigma \frac{\partial \vec{e}}{\partial t} = 0$$

$$e = e_0 e^{-i(kz - \omega t)} + e_0 e^{i(kz + \omega t)}$$

$$\frac{\partial^2 \vec{h}}{\partial z^2} - \mu\sigma \frac{\partial \vec{h}}{\partial t} = 0$$

$$h = h_0 e^{-i(kz - \omega t)} + h_0 e^{i(kz + \omega t)}$$

onde k é uma grandeza complexa:

$$\frac{\partial^2 \vec{E}}{\partial z^2} - i\mu\sigma\omega\vec{E} = 0$$

$k = \alpha - i\beta$ (com α e β números reais; Stratton, 1941).

$$\frac{\partial^2 \vec{H}}{\partial z^2} - i\mu\sigma\omega\vec{H} = 0$$

α e β são dados por (Stratton, 1941).

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Métodos eletromagnéticos

$$\alpha = \omega \left(\frac{\mu \varepsilon}{2} \left(\left(1 + \frac{\sigma^2}{\varepsilon^2 \omega^2} \right)^{1/2} + 1 \right) \right)^{1/2}$$

$$\beta = \omega \left(\frac{\mu \varepsilon}{2} \left(\left(1 + \frac{\sigma^2}{\varepsilon^2 \omega^2} \right)^{1/2} - 1 \right) \right)^{1/2}$$

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Métodos eletromagnéticos

- Quando as correntes de condução dominam sobre as de deslocamento, como é o caso das aplicações que serão aqui enfocadas, α e β são idênticos (Nabighian & Macnae, 1987a):

$$\left(\frac{\sigma^2}{\varepsilon^2 \omega^2} \gg 1 \right)$$

$$\alpha = \beta = \left(\frac{\omega \mu \sigma}{2} \right)^{1/2}$$

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Métodos eletromagnéticos

- $e = e_0 e^{-i(kz - \omega t)} + e_0 e^{i(kz + \omega t)}$
- $h = h_0 e^{-i(kz - \omega t)} + h_0 e^{i(kz + \omega t)}$

- substituindo!!!

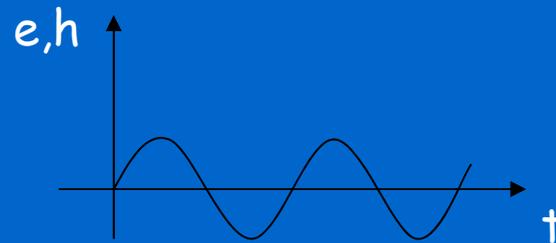
- $e = e_0 e^{-i\alpha z} e^{-\beta z} e^{i\omega t}$
- $h = h_0 e^{-i\alpha z} e^{-\beta z} e^{i\omega t}$

Métodos eletromagnéticos

- Assim conclui-se que:
- 1 - $e^{-\beta z}$ fator de atenuação com a distância (REAL);
- A amplitude da onda eletromagnética decai de um fator de $1/e$ com a distância;

$$\delta = \left(\frac{2}{\omega \mu \sigma} \right)^{1/2} = 503 \left(\frac{1}{f \sigma} \right)^{1/2}$$

- ; onde δ é a profundidade de penetração.
- 2 - onda varia senoidalmente com z ; $e^{-i\alpha z} = \cos(\alpha z) - i \sin(\alpha z)$
- 3 - onda varia senoidalmente com t ; $e^{i\omega t} = \cos(\omega t) - i \sin(\omega t)$
- 4 - e e h variam com o tempo, de acordo com o seguinte gráfico:



- 5 - Se a propagação da onda tiver direção z , e e h variam senoidalmente com z .

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Métodos eletromagnéticos

- Assim conclui-se que:
- $1 - e^{-\beta z}$ fator de atenuação com a distância (REAL);
- A amplitude da onda eletromagnética decai de um fator de $1/e$ com a distância;

$$\delta = \left(\frac{2}{\omega \mu \sigma} \right)^{1/2} = 503 \left(\frac{1}{f \sigma} \right)^{1/2}$$

Métodos eletromagnéticos

- Os levantamentos e.m. não requerem contacto físico do receptor/emissor com o solo, podendo ser feitos de avião.

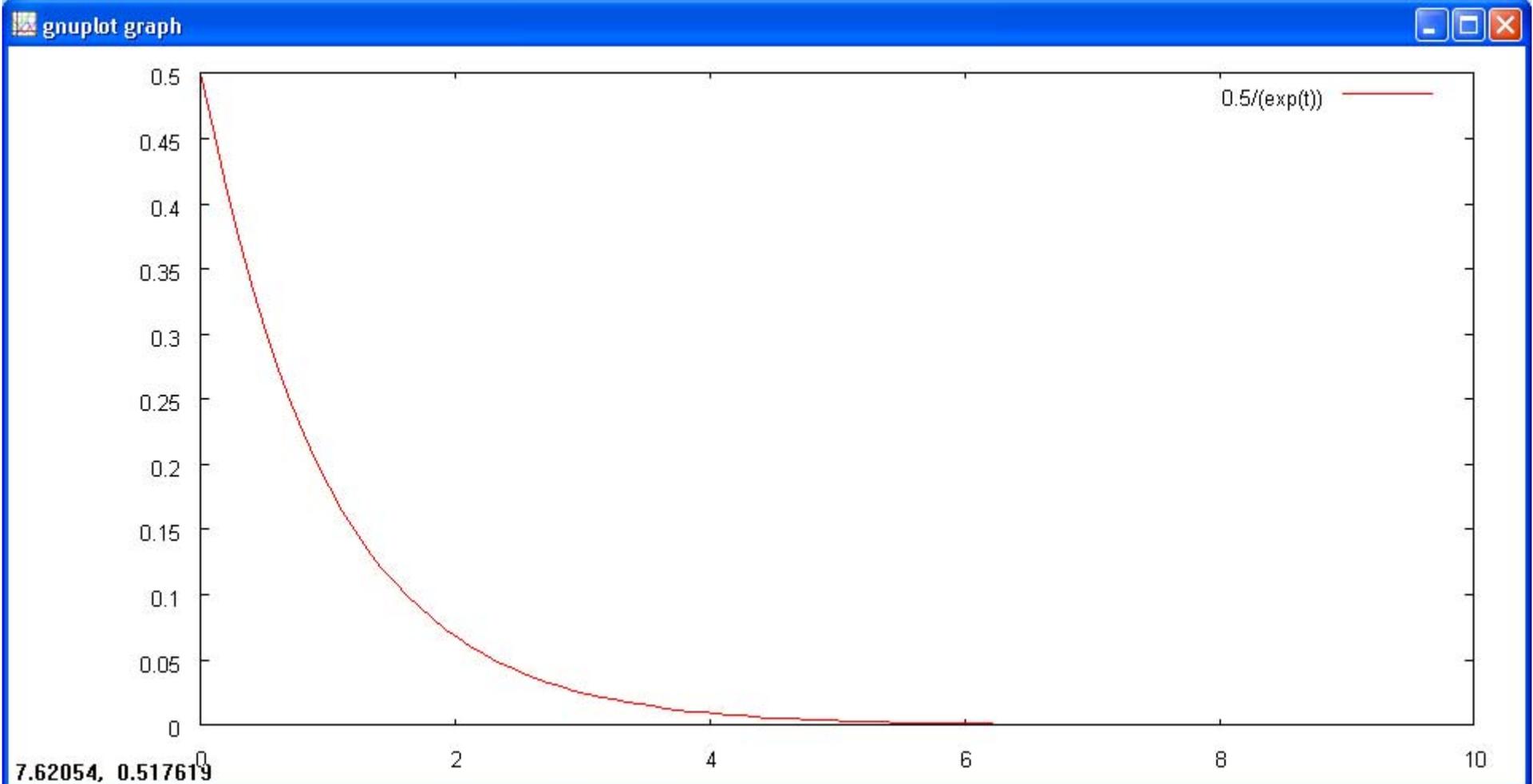
A profundidade de penetração de uma onda electromagnética depende

- da sua frequência
- da condutividade eléctrica ($\sigma=1/\rho$) do meio

A profundidade de penetração, d , pode ser definida como a profundidade para a qual a amplitude do campo, A_z , decresce por um factor $1/e$ (comparada com a amplitude à superfície, A_0):

$$A_z = A_0 e^{-1}$$

Métodos eletromagnéticos



A_0 :

$$Az = A_0 e^{-1}$$

Métodos eletromagnéticos

$$e = e_0 e^{-i\alpha z} e^{-\beta z} e^{i\omega t}$$

- Num plano - fase constante é descrita por:

$$e = e_0 e^{-i(\alpha z - \omega t)} = e_0 e^{-ic}$$

- Onde c é a fase, ie descrição da amplitude da onda senoidal em função de z e t .
- Se tivermos

$$\alpha z - \omega t = c$$

$$\frac{dz}{dt} = \frac{\omega}{\alpha} = V_{\text{fase}}$$

Métodos eletromagnéticos

$$e = e_0 e^{-\alpha z} e^{-\beta z} e^{i\omega t}$$

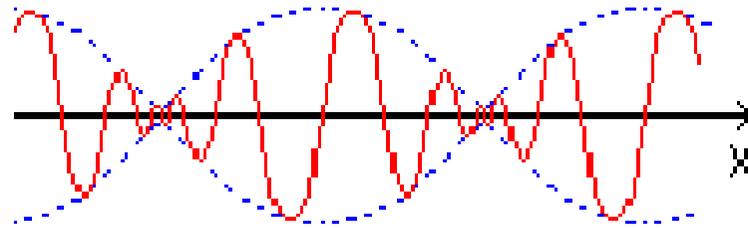
- Um meio é caracterizado como dispersivo se a velocidade de fase é em função da frequência, ie,

$$V_{\text{fase}} = V_{\text{fase}}(\omega)$$

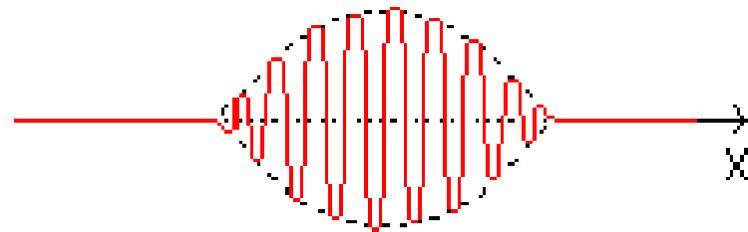
- Num meio dispersivo, as ondas de diferentes frequências que compõe o pulso têm velocidades de módulos diferentes. O módulo da velocidade do pulso pode não ser igual ao módulo das velocidades de fase.
- Num meio não dispersivo, a informação viaja à velocidade de fase, que é idêntica à velocidade de grupo. Num meio dispersivo, a informação viaja à velocidade de grupo.

Métodos eletromagnéticos

- Num método de pulso, a velocidade de fase.
- Num método de fase, a



(a)



(b)

que compõe
o da
ocidades de

de fase,
vo, a

Métodos eletromagnéticos

$$\mathbf{e} = e_0 e^{-i\alpha z} e^{-\beta z} e^{i\omega t}$$

- Similarmente o número de onda é função de ω , ie .

$$\mu\epsilon\omega^2 - i\mu\sigma\omega = k^2$$

$$\omega = \omega(k)$$

- Lembrar que a corrente de deslocamento é negligenciada correspondendo a:

$$\alpha = \beta = \left(\frac{\omega\mu\sigma}{2} \right)^{1/2}$$

- ou aproximação quase estática

$$\frac{dz}{dt} = \frac{\omega}{\alpha} = V_{fase}$$

$$V_{fase} = \sqrt{\frac{2\omega}{\mu\sigma}}$$

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Métodos eletromagnéticos

- A segunda solução básica das equações de onda é para os campos elétrico e magnético causados por um impulso, no plano $z = 0$. Se for negligenciada a corrente de deslocamento, as soluções se darão utilizando a função de Green (Nabighian & Macnae, 1987a).

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Métodos eletromagnéticos

2ª. Solução: por analogia com a função de Green – p 171 Nabighian

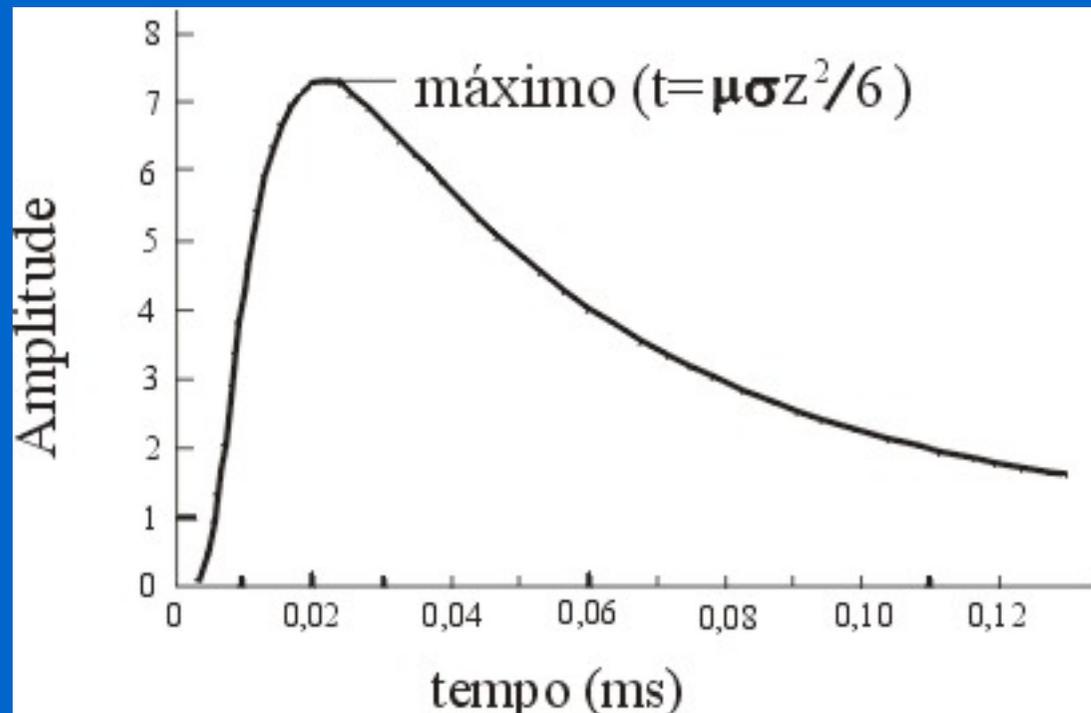
$$\frac{\partial^2 \vec{e}}{\partial z^2} - \mu\sigma \frac{\partial \vec{e}}{\partial t} = 0 \qquad h = h_0 \frac{(\mu\sigma)^{\frac{1}{2}} z}{2\pi^{\frac{1}{2}} t^{\frac{3}{2}}} e^{-\mu\sigma z^2 / 4t}$$

$$\frac{\partial^2 \vec{h}}{\partial z^2} - \mu\sigma \frac{\partial \vec{h}}{\partial t} = 0 \qquad e = e_0 \frac{(\mu\sigma)^{\frac{1}{2}} z}{2\pi^{\frac{1}{2}} t^{\frac{3}{2}}} e^{-\mu\sigma z^2 / 4t}$$

Métodos eletromagnéticos

- A Figura abaixo mostra o campo magnético em função do tempo. Observa-se que ele é máximo onde:

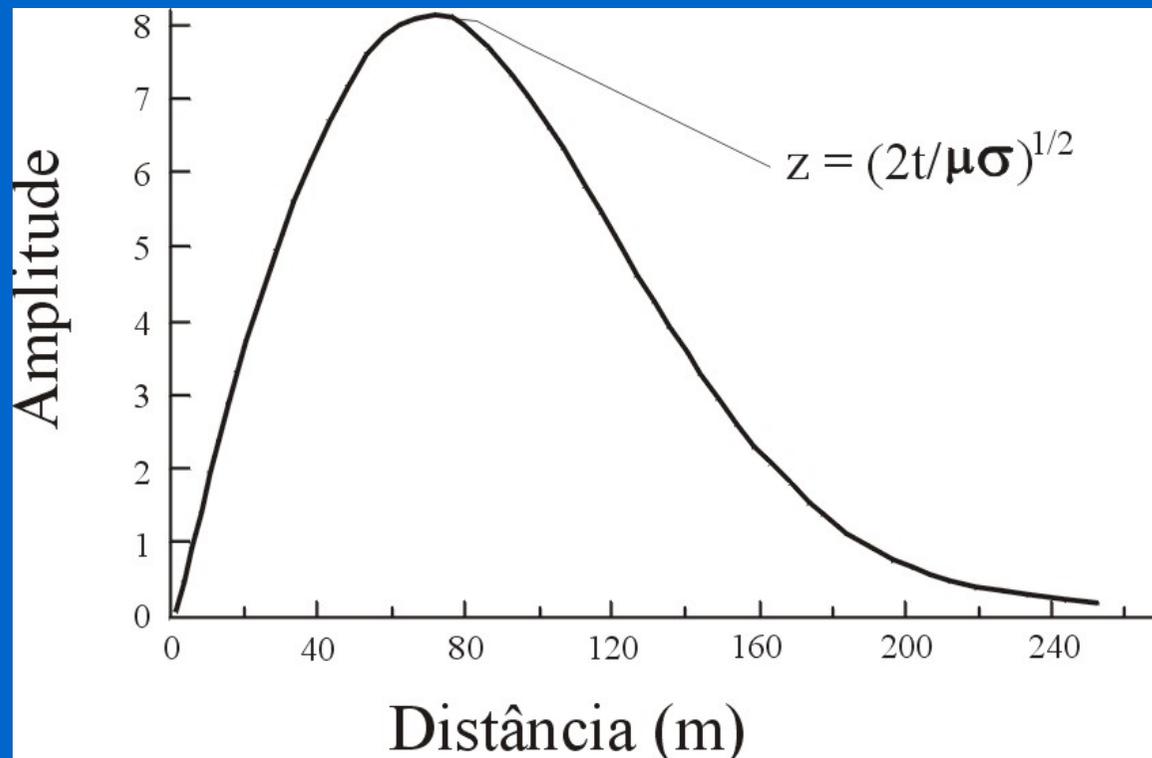
$$h = h_0 \frac{(\mu\sigma)^{\frac{1}{2}} z}{2\pi^{\frac{1}{2}} t^{\frac{3}{2}}} e^{-\mu\sigma z^2 / 4t}$$



Métodos eletromagnéticos

- A Figura abaixo mostra o campo em função da distância z (penetração) para um tempo fixo, e ele é máximo para

$$h = h_0 \frac{(\mu\sigma)^{\frac{1}{2}} z}{2\pi^{\frac{1}{2}} t^{\frac{3}{2}}} e^{-\mu\sigma z^2 / 4t}$$



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Métodos eletromagnéticos

- Nota-se uma similaridade entre a profundidade z e *skin deph* (obtida nas formulações no domínio da frequência -) pois:

$$z \propto t^{\frac{1}{2}}$$

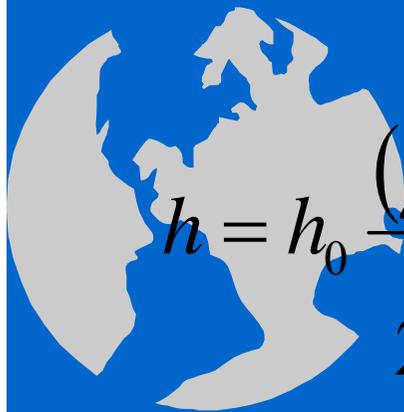
- e o skin deph $\propto \omega^{-\frac{1}{2}}$
- Assim, de acordo com a frequência utilizada pelo sistema e da condutividade do meio em estudo, tem-se uma idéia da profundidade que se está investigando (Ward & Hohmann, 1988).

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T. de Geofísica: Introdução ao Método eletromagnético

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Resumo

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Soluções da onda

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• $e = e_0 e^{-i\alpha z} e^{-\beta z} e^{i\omega t}$
• $h = h_0 e^{-i\alpha z} e^{-\beta z} e^{i\omega t}$



$$h = h_0 \frac{(\mu\sigma)^{\frac{1}{2}} z}{2\pi^{\frac{1}{2}} t^{\frac{3}{2}}} e^{-\mu\sigma z^2 / 4t}$$

$$e = e_0 \frac{(\mu\sigma)^{\frac{1}{2}} z}{2\pi^{\frac{1}{2}} t^{\frac{3}{2}}} e^{-\mu\sigma z^2 / 4t}$$

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Domínio da frequência

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Impedância mútua Z:

Z = acoplamento mútuo T e R na presença da terra

Z₀ = acoplamento mútuo T e R no espaço

H^P: campo primário


$$\frac{Z}{Z_0} = \frac{H}{H^P}$$

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Domínio da frequência

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Impedância mútua: razão da tensão induzida no receptor pela corrente transmitida no transmissor (fonte)

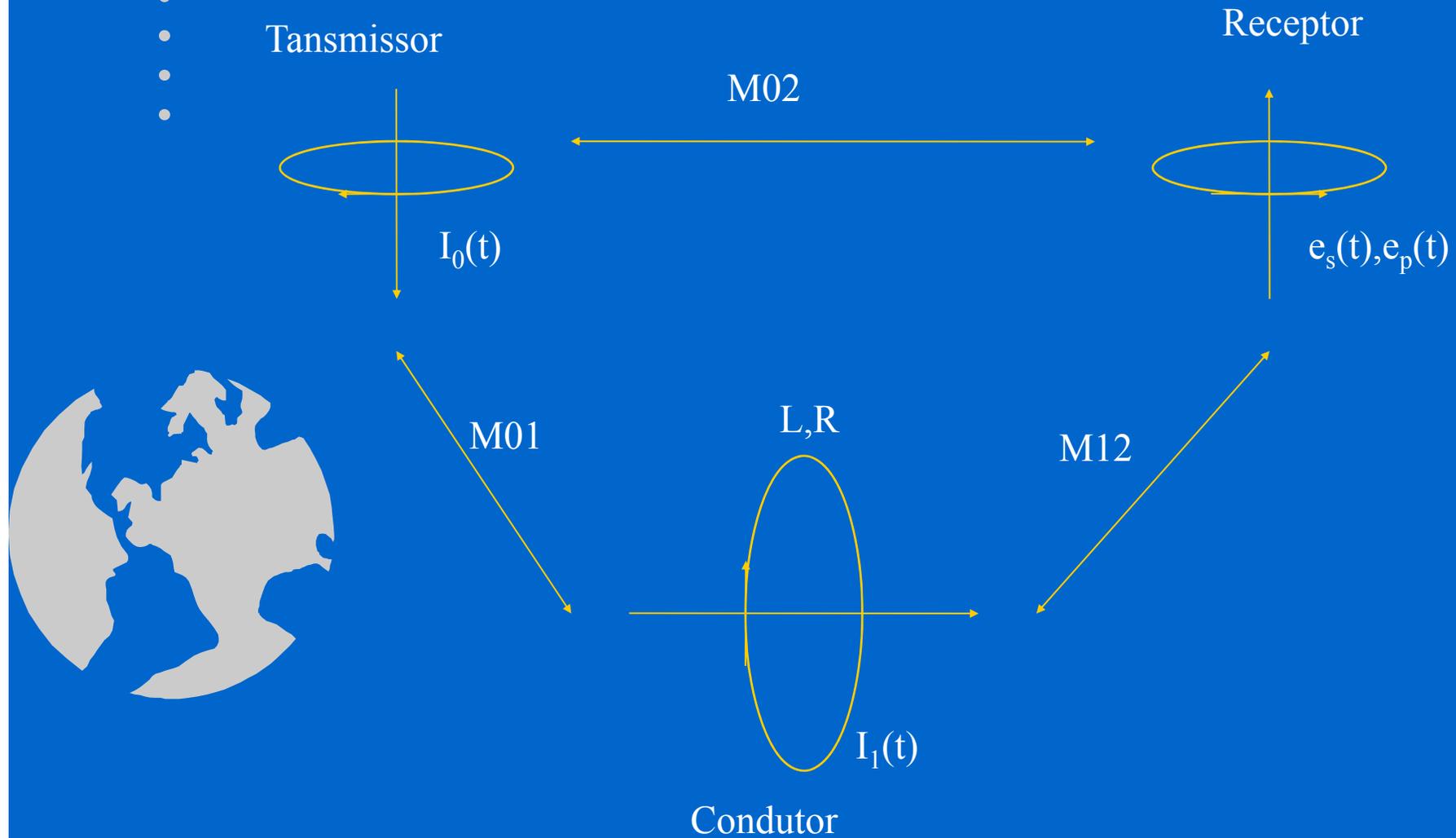
m: momento ou produto área x no. de voltas do receptor

H: campo no receptor



$$Z = \frac{V}{I} = \frac{-i\mu_0 \omega m H}{I}$$

Métodos eletromagnéticos



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 • T. de Geofísica: Introdução ao Método eletromagnético
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 • **Domínio da frequência**
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 • **Fields on the surface of a homogeneous half-space in the frequency domain.**

Free-Space Value	Mutual Coupling Ratio
Vertical Magnetic Dipole Source	
$E_{\phi}^P = \frac{-i\mu_0\omega m}{4\pi r^2}$	$\frac{Z}{Z_0} = \frac{2}{k^2 r^2} [(3 + 3ikr - k^2 r^2)e^{-ikr} - 3]$
$H_z^P = \frac{-m}{4\pi r^3}$	$\frac{Z}{Z_0} = \frac{2}{k^2 r^2} [(9 + 9ikr - 4k^2 r^2 - ikr^3 r^3)e^{-ikr} - 9]$
$H_r^P = 0$	$\frac{Z}{Z_0} = k^2 r^2 \left[I_1\left(\frac{ikr}{2}\right) K_1\left(\frac{ikr}{2}\right) - I_2\left(\frac{ikr}{2}\right) K_2\left(\frac{ikr}{2}\right) \right]$ where Z_0 (perp) is set = Z_0 (HCP)

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Domínio da frequência

Antena - bobina

Bobinas horizontais - coplanares

Bobinas perpendiculares

High-Frequency Asymptote (far zone, $|kr| \gg 1$)

Vertical Magnetic Dipole Source

$$\frac{Z^h}{Z_0} = \frac{-6}{k^2 r^2}$$

$$\frac{Z^h}{Z_0} = \frac{-18}{k^2 r^2}$$

$$\frac{Z^h}{Z_0} = \frac{-6}{ikr}$$

$$\rho_a^h = \frac{\mu_0 \omega r^2}{6} \operatorname{Im} \left(\frac{Z}{Z_0} \right)$$

$$\rho_a^h = \frac{\mu_0 \omega r^2}{18} \operatorname{Im} \left(\frac{Z}{Z_0} \right)$$

$$\rho_a^h = \frac{\mu_0 \omega r^2}{36} \operatorname{Im} \left[\left(\frac{Z}{Z_0} \right)^2 \right]$$

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Domínio da frequência

Table 1. Continued

Low-Frequency Asymptote (near zone, $|kr| \ll 1$)

Vertical Magnetic Dipole Source

$$\frac{Z^\ell}{Z_0} = 1 + \frac{k^2 r^2}{4}$$

$$\frac{Z^\ell}{Z_0} = 1 - \frac{k^2 r^2}{4}$$

$$\frac{Z^\ell}{Z_0} = \frac{k^2 r^2}{4}$$

$$\rho_a^\ell = \frac{-\mu_0 \omega r^2}{4} \left[\text{Im} \left(\frac{Z}{Z_0} \right) \right]^{-1}$$

$$\rho_a^\ell = \frac{\mu_0 \omega r^2}{4} \left[\text{Im} \left(\frac{Z}{Z_0} \right) \right]^{-1}$$

$$\rho_a^\ell = \frac{-\mu_0 \omega r^2}{4} \left[\text{Im} \left(\frac{Z}{Z_0} \right) \right]^{-1}$$

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Domínio da frequência



Table 1. Quasi-static fields of various sources on the surface of a homogeneous half-space in the frequency domain.

General Expression	Free-Space Value	Mutual Coupling Ratio
Vertical Magnetic Dipole Source		
$E_\phi = \frac{-m}{2\pi\sigma r^4} [3 - (3 + 3ikr - k^2r^2)e^{-ikr}]$ (loop-wire)	$E_\phi^f = \frac{-i\mu_0\omega m}{4\pi r^2}$	$\frac{Z}{Z_0} = \frac{2}{k^2r^2} [(3 + 3ikr - k^2r^2)e^{-ikr} - 3]$
$H_z = \frac{-m}{2\pi k^2 r^3} [(9 + 9ikr - 4k^2r^2 - ik^3r^3)e^{-ikr} - 9]$ (horizontal coplanar loops)	$H_z^f = \frac{-m}{4\pi r^3}$	$\frac{Z}{Z_0} = \frac{2}{k^2r^2} [(9 + 9ikr - 4k^2r^2 - ik^3r^3)e^{-ikr} - 9]$
$H_r = \frac{-mk^2}{4\pi r} \left[I_1\left(\frac{ikr}{2}\right)K_1\left(\frac{ikr}{2}\right) - I_2\left(\frac{ikr}{2}\right)K_2\left(\frac{ikr}{2}\right) \right]$ (perpendicular loops)	$H_r^f = 0$	$\frac{Z}{Z_0} = k^2r^2 \left[I_1\left(\frac{ikr}{2}\right)K_1\left(\frac{ikr}{2}\right) - I_2\left(\frac{ikr}{2}\right)K_2\left(\frac{ikr}{2}\right) \right]$ where Z_0 (perp) is set = Z_0 (HCP)
Horizontal Magnetic Dipole Source (x-directed)		
$H_\phi = \frac{-m}{2\pi k^2 r^3} [3 + k^2r^2 - (3 + 3ikr - k^2r^2)e^{-ikr}]$	$H_\phi^f = \frac{-m}{4\pi r^3}$	$\frac{Z}{Z_0} = \frac{2}{k^2r^2} [3 + k^2r^2 - (3 + 3ikr - k^2r^2)e^{-ikr}]$ (vertical coplanar loops)
$H_r = \frac{m}{2\pi k^2 r^3} [12 + 2k^2r^2 - (12 + 12ikr - 5k^2r^2 - ik^3r^3)e^{-ikr}]$ (vertical coaxial loops)	$H_r^f = \frac{m}{2\pi r^3}$	$\frac{Z}{Z_0} = \frac{1}{k^2r^2} [12 + 2k^2r^2 - (12 + 12ikr - 5k^2r^2 - ik^3r^3)e^{-ikr}]$
Horizontal Electric Dipole Source (x-directed)		
$E_x = E_r = \frac{P}{2\pi\sigma r^3} [1 + (1 + ikr)e^{-ikr}]$ (in-line electric dipole-dipole)	$E_x^f = \frac{P}{\pi\sigma r^3}$ (for homog. half-space)	$\frac{Z}{Z_0} = \frac{1}{2} [1 + (1 + ikr)e^{-ikr}]$
$E_x = -E_\phi = \frac{-P}{2\pi\sigma r^3} [2 - (1 + ikr)e^{-ikr}]$ (equatorial electric dipole-dipole)	$E_x^f = \frac{-P}{2\pi\sigma r^3}$ (for homog. half-space)	$\frac{Z}{Z_0} = 2 - (1 + ikr)e^{-ikr}$
$H_r = \frac{P}{2\pi r^2} \left[3I_1K_1 - \frac{ikr}{2}(I_0K_1 - I_1K_0) \right]$ where I_1, K_1 have arguments $(ikr/2)$ (wire-loop, equatorial configuration)	$H_r^f = \frac{P}{4\pi r^2}$	$\frac{Z}{Z_0} = 2 \left[3I_1K_1 - \frac{ikr}{2}(I_0K_1 - I_1K_0) \right]$ where I_1, K_1 have argument $(ikr/2)$
$H_\phi = \frac{-P}{2\pi r^2} I_1\left(\frac{ikr}{2}\right)K_1\left(\frac{ikr}{2}\right)$ (wire-loop, in-line configuration)	$H_\phi^f = \frac{-P}{4\pi r^2}$	$\frac{Z}{Z_0} = 2I_1\left(\frac{ikr}{2}\right)K_1\left(\frac{ikr}{2}\right)$
$H_z = \frac{-P}{2\pi k^2 r^4} [3 - (3 + 3ikr - k^2r^2)e^{-ikr}]$ (equatorial configuration)	$H_z^f = \frac{P}{4\pi r^2}$	$\frac{Z}{Z_0} = \frac{-2}{k^2r^2} [3 - (3 + 3ikr - k^2r^2)e^{-ikr}]$
Large Horizontal Loop		
$H_z = \frac{-I}{k^2a^3} [3 - (3 + 3ika - k^2a^2)e^{-ika}]$ (central loop (in-loop) configuration)	$H_z^f = \frac{I}{2a}$	$\frac{Z}{Z_0} = \frac{-2}{k^2a^2} [3 - (3 + 3ika - k^2a^2)e^{-ika}]$
Line Source (x-directed)		
$E_x = \frac{-I}{\pi\sigma y^2} [1 - ikyK_1(iky)]$	$E_x^f = \frac{-\omega\mu_0 I}{\pi} K_0(iky)$	$\frac{Z}{Z_0} = 1 - iky K_1(iky)$ where Z_0 is set = Z_0 (E_x , high-freq asymptote)
$H_z = \frac{-I}{\pi k^2 y^3} [2ikyK_1(iky) - k^2y^2K_0(iky) - 2]$	$H_z^f = \frac{-I}{2\pi y}$	$\frac{Z}{Z_0} = \frac{2}{k^2y^2} [2ikyK_1(iky) - k^2y^2K_0(iky) - 2]$
$H_y = \frac{Ik}{\pi} \left\{ \frac{1}{3} + \frac{\pi}{2} \left[\frac{J_2(-ky)}{iky} + \frac{H_2(-ky)}{ky} \right] \right\}$	$H_y^f = 0$	$\frac{Z}{Z_0} = 2iky \left\{ \frac{1}{3} + \frac{\pi}{2} \left[\frac{J_2(-ky)}{iky} + \frac{H_2(-ky)}{ky} \right] \right\}$

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Domínio da frequência



Table 1. Continued

High-Frequency Asymptote (far zone, $ikr \gg 1$)

Vertical Magnetic Dipole Source		
$E_{\phi}^h = \frac{-3m}{2\pi\sigma r^4}$	$\frac{Z^h}{Z_0} = \frac{-6}{k^2 r^2}$	$\rho_a^h = \frac{\mu_0 \omega r^2}{6} \operatorname{Im} \left(\frac{Z}{Z_0} \right)$
$H_z^h = \frac{9m}{2\pi k^2 r^5}$	$\frac{Z^h}{Z_0} = \frac{-18}{k^2 r^2}$	$\rho_a^h = \frac{\mu_0 \omega r^2}{18} \operatorname{Im} \left(\frac{Z}{Z_0} \right)$
$H_r^h = \frac{3m}{2\pi i k r^4}$	$\frac{Z^h}{Z_0} = \frac{-6}{i k r}$	$\rho_a^h = \frac{\mu_0 \omega r^2}{36} \operatorname{Im} \left[\left(\frac{Z}{Z_0} \right)^2 \right]$
Horizontal Magnetic Dipole Source (x-directed)		
$H_{\phi}^h = \frac{-m}{2\pi r^3} \left[1 + \frac{3}{k^2 r^2} \right]$	$\frac{Z^h}{Z_0} = 2 + \frac{6}{k^2 r^2}$	$\rho_a^h = \frac{-\mu_0 \omega r^2}{6} \operatorname{Im} \left(\frac{Z}{Z_0} \right)$
$H_z^h = \frac{-m}{2\pi r^3} \left[2 + \frac{12}{k^2 r^2} \right]$	$\frac{Z^h}{Z_0} = 2 + \frac{12}{k^2 r^2}$	$\rho_a^h = \frac{-\mu_0 \omega r^2}{12} \operatorname{Im} \left(\frac{Z}{Z_0} \right)$
Horizontal Electric Dipole Source (x-directed)		
$E_x^h = \frac{p}{2\pi\sigma r^3}$	$\frac{Z^h}{Z_0} = \frac{1}{2}$	$\rho_a^h = \frac{2\pi r^3}{p} E_x$
$E_z^h = \frac{-p}{\pi\sigma r^3}$	$\frac{Z^h}{Z_0} = 2$	$\rho_a^h = \frac{-\pi r^3}{p} E_x$
$H_{\phi}^h = \frac{p}{\pi i k r^3} \left(1 + \frac{3}{k^2 r^2} \right)$	$\frac{Z^h}{Z_0} = \frac{4}{i k r} \left(1 + \frac{3}{k^2 r^2} \right)$	$\rho_a^h = \frac{\mu_0 \omega r^2}{16} \left[\operatorname{Im} \left(\frac{Z}{Z_0} \right) \right]^2$
$H_z^h = \frac{-p}{2\pi i k r^3} \left(1 + \frac{3}{2k^2 r^2} \right)$	$\frac{Z^h}{Z_0} = \frac{2}{i k r} \left(1 + \frac{3}{2k^2 r^2} \right)$	$\rho_a^h = \frac{\mu_0 \omega r^2}{4} \left[\operatorname{Im} \left(\frac{Z}{Z_0} \right) \right]^2$
$H_r^h = \frac{-3p}{2\pi k^2 r^4}$	$\frac{Z^h}{Z_0} = \frac{-6}{k^2 r^2}$	$\rho_a^h = \frac{\mu_0 \omega r^2}{6} \operatorname{Im} \left(\frac{Z}{Z_0} \right)$
Large Horizontal Loop		
$H_z^h = \frac{-3I}{k^2 a^3}$	$\frac{Z^h}{Z_0} = \frac{-6}{k^2 a^2}$	$\rho_a^h = \frac{\mu_0 \omega a^2}{6} \operatorname{Im} \left(\frac{Z}{Z_0} \right)$
Line Source (x-directed)		
$E_z^h = \frac{-I}{\pi \sigma y^2} \left[1 - \left(\frac{\pi i k y}{2} \right)^{1/2} e^{-iky} \right]$	$\frac{Z^h}{Z_0} = 1 - \left(\frac{\pi i k y}{2} \right)^{1/2} e^{-iky}$	$\rho_a^h = \frac{-\pi y^2 E_x}{I}$
$H_z^h = \frac{-I}{\pi k^2 y^3} \left[\left(\frac{\pi}{2} \right)^{1/2} (iky)^{3/2} e^{-iky} - 2 \right]$	$\frac{Z^h}{Z_0} = \frac{2}{k^2 y^3} \left[\left(\frac{\pi}{2} \right)^{1/2} (iky)^{3/2} e^{-iky} - 2 \right]$	$\rho_a^h = \frac{\mu_0 \omega y^2}{4} \operatorname{Im} \left(\frac{Z}{Z_0} \right)$
$H_y^h = \frac{I}{\pi i k y^2}$	$\frac{Z^h}{Z_0} = \frac{-2}{iky}$	$\rho_a^h = \frac{\mu_0 \omega y^2}{4} \left[\operatorname{Im} \left(\frac{Z}{Z_0} \right) \right]^2$

T. de Geofísica: Introdução ao Método eletromagnético

Domínio da frequência



Table 1. Continued

Low-Frequency Asymptote (near zone, $ikr \ll 1$)

$$E_{\phi}^{\epsilon} = \frac{m}{4\pi\sigma r^4} \left(k^2 r^2 + \frac{k^4 r^4}{4} \right)$$

$$H_z^{\epsilon} = \frac{-m}{4\pi r^3} \left(1 - \frac{k^2 r^2}{4} \right)$$

$$H_{\phi}^{\epsilon} = \frac{-mk^2}{16\pi r}$$

$$H_{\phi}^{\epsilon} = \frac{-m}{4\pi r^3} \left[1 - \frac{k^2 r^2}{4} \right]$$

$$H_z^{\epsilon} = \frac{m}{2\pi r^3} \left[1 - \frac{ik^3 r^5}{15} \right]$$

$$E_x^{\epsilon} = \frac{p}{2\pi\sigma r^3} \left[2 + \frac{k^2 r^2}{2} \right]$$

$$E_z^{\epsilon} = \frac{-p}{2\pi\sigma r^3} \left[1 - \frac{k^2 r^2}{2} \right]$$

$$H_z^{\epsilon} = \frac{p}{4\pi r^2} \left[1 + \frac{k^2 r^2}{8} \left(\ln \frac{ikr}{4} + C + \frac{3}{4} \right) \right]$$

$$H_{\phi}^{\epsilon} = \frac{-p}{4\pi r^2} \left[1 - \frac{k^2 r^2}{8} \left(\ln \frac{ikr}{4} + C - \frac{1}{4} \right) \right]$$

$$H_z^{\epsilon} = \frac{p}{4\pi k^2 r^4} \left[k^2 r^2 + \frac{k^4 r^4}{4} \right]$$

$$H_z^{\epsilon} = \frac{I}{2k^2 a^3} \left[k^2 a^2 + \frac{k^4 a^4}{4} \right]$$

$$E_x^{\epsilon} = \frac{-Ik^2}{2\pi\sigma} \left(\ln \frac{iky}{2} + C - \frac{1}{2} \right)$$

$$H_z^{\epsilon} = \frac{-I}{2\pi y} \left[1 - \frac{k^2 y^2}{4} \left(\ln \frac{iky}{2} + C - \frac{3}{4} \right) \right]$$

$$H_y^{\epsilon} = \frac{Ik}{3\pi}$$

Vertical Magnetic Dipole Source

$$\frac{Z^{\epsilon}}{Z_0} = 1 + \frac{k^2 r^2}{4}$$

$$\frac{Z^{\epsilon}}{Z_0} = 1 - \frac{k^2 r^2}{4}$$

$$\frac{Z^{\epsilon}}{Z_0} = \frac{k^2 r^2}{4}$$

Horizontal Magnetic Dipole Source (x-directed)

$$\frac{Z^{\epsilon}}{Z_0} = 1 - \frac{k^2 r^2}{4}$$

$$\frac{Z^{\epsilon}}{Z_0} = 1 - \frac{ik^3 r^3}{15}$$

Horizontal Electric Dipole Source (x-directed)

$$\frac{Z^{\epsilon}}{Z_0} = 1 + \frac{k^2 r^2}{4}$$

$$\frac{Z^{\epsilon}}{Z_0} = 1 - \frac{k^2 r^2}{2}$$

$$\frac{Z^{\epsilon}}{Z_0} = 1 + \frac{k^2 r^2}{8} \left(\ln \frac{ikr}{4} + C + \frac{3}{4} \right)$$

$$\frac{Z^{\epsilon}}{Z_0} = 1 - \frac{k^2 r^2}{8} \left(\ln \frac{ikr}{4} + C - \frac{1}{4} \right)$$

$$\frac{Z^{\epsilon}}{Z_0} = 1 + \frac{k^2 r^2}{4}$$

Large Horizontal Loop

$$\frac{Z^{\epsilon}}{Z_0} = 1 + \frac{k^2 a^2}{4}$$

Line Source (x-directed)

$$\frac{Z^{\epsilon}}{Z_0} = \frac{k^2 y^2}{2} \left(\ln \frac{iky}{2} + C - \frac{1}{2} \right)$$

$$\frac{Z^{\epsilon}}{Z_0} = \left[1 - \frac{k^2 y^2}{4} \left(\ln \frac{iky}{2} + C - \frac{3}{4} \right) \right]$$

$$\frac{Z^{\epsilon}}{Z_0} = \frac{-2iky}{3}$$

$$\rho_a^{\epsilon} = \frac{-\mu_0 \omega r^2}{4} \left[\text{Im} \left(\frac{Z}{Z_0} \right) \right]^{-1}$$

$$\rho_a^{\epsilon} = \frac{\mu_0 \omega r^2}{4} \left[\text{Im} \left(\frac{Z}{Z_0} \right) \right]^{-1}$$

$$\rho_a^{\epsilon} = \frac{-\mu_0 \omega r^2}{4} \left[\text{Im} \left(\frac{Z}{Z_0} \right) \right]^{-1}$$

$$\rho_a^{\epsilon} = \frac{\mu_0 \omega r^2}{4} \left[\text{Im} \left(\frac{Z}{Z_0} \right) \right]^{-1}$$

$$\rho_a^{\epsilon} = \frac{\mu_0 \omega r^2}{\left[15 \text{Im} \left(\frac{Z}{Z_0} \right) \right]^{2/3}}$$

$$\rho_a^{\epsilon} = \frac{-\mu_0 \omega r^2}{4} \left[\text{Im} \left(\frac{Z}{Z_0} \right) \right]^{-1}$$

or $\rho_a = \frac{\pi r^3}{p} E_x$

$$\rho_a^{\epsilon} = \frac{\mu_0 \omega r^2}{2} \left[\text{Im} \left(\frac{Z}{Z_0} \right) \right]^{-1}$$

or $\rho_a^{\epsilon} = \frac{-2\pi r^3}{p} E_x$

$$\rho_a^{\epsilon} = \frac{\pi \mu_0 \omega r^2}{32 \left(\text{Re} \frac{Z}{Z_0} - 1 \right)}$$

$$\rho_a^{\epsilon} = \frac{\pi \mu_0 \omega r^2}{32 \left(1 - \text{Re} \frac{Z}{Z_0} \right)}$$

$$\rho_a^{\epsilon} = \frac{-\mu_0 \omega r^2}{4} \left[\text{Im} \left(\frac{Z}{Z_0} \right) \right]^{-1}$$

$$\rho_a^{\epsilon} = \frac{-\mu_0 \omega a^2}{4} \left[\text{Im} \left(\frac{Z}{Z_0} \right) \right]^{-1}$$

$$\rho_a^{\epsilon} = \frac{\pi \mu_0 \omega y^2}{8 \text{Re} \left(\frac{Z}{Z_0} \right)}$$

$$\rho_a^{\epsilon} = \frac{\pi \mu_0 \omega y^2}{16 \left[\text{Re} \left(1 - \frac{Z}{Z_0} \right) \right]}$$

$$\rho_a^{\epsilon} = \frac{4\mu_0 \omega y^2}{9 \left[\text{Im} \left(\frac{Z}{Z_0} \right) \right]^2}$$

Métodos eletromagnéticos

- Bobina Horizontal – in loop

General Expression	D.C. Value ($t < 0$)
$h_z = \frac{It}{\sigma\mu_0 a^3} \left[(2\theta^2 a^2 - 3) \operatorname{erf}(\theta a) + \frac{6}{\sqrt{\pi}} \theta a e^{-\theta^2 a^2} \right]$ <p>(central loop; in loop)</p>	$h_z^0 = \frac{I}{2a}$
$\frac{\partial h_z}{\partial t} = \frac{-I}{\sigma\mu_0 a^3} \left[3 \operatorname{erf}(\theta a) - \frac{2}{\sqrt{\pi}} \theta a (3 + 2\theta^2 a^2) e^{-\theta^2 a^2} \right]$	

Métodos eletromagnéticos

- Bobina Horizontal – in loop

Early-Time Asymptote
($\theta r \gg 1, t \rightarrow 0$)

Large Horizontal Loop

$$h_z^e = \frac{I}{2a} \left[1 - \frac{6t}{\sigma \mu_0 a^2} \right] \quad \rho_a^e = \frac{\mu_0 a^3}{3It} \left(\frac{I}{2a} - h_z \right)$$

$$\frac{\partial h_z^e}{\partial t} = \frac{-3I}{\sigma \mu_0 a^3} \quad \rho_a^e = \frac{-\mu_0 a^3}{3I} \frac{\partial h_z}{\partial t}$$

Late-Time Asymptote
($\theta r \ll 1, t \rightarrow \infty$)

$$h_z^\ell = \frac{I \sigma^{3/2} \mu_0^{3/2} a^2}{30 \pi^{1/2} t^{3/2}} \quad \rho_a^\ell = \frac{I^{2/3} \mu_0^{2/3} 2^{4/3}}{30^{2/3} \pi^{1/3} t} (h_z)^{-2/3}$$

$$\frac{\partial h_z^\ell}{\partial t} = \frac{-I \sigma^{3/2} \mu_0^{3/2} a^2}{20 \pi^{1/2} t^{5/2}} \quad \rho_a^\ell = \frac{I^{2/3} \mu_0^{4/3}}{20^{2/3} \pi^{1/3} t^{5/3}} \left(\frac{-\partial h_z}{\partial t} \right)$$

Métodos eletromagnéticos

Table 2. Quasistatic fields of various sources on the surface of a homogeneous half-space in the time domain.

General Expression	D.C. Value ($t < 0$)	Early-Time Asymptote ($\theta r \gg 1, t \rightarrow 0$)		Late-Time Asymptote ($\theta r \ll 1, t \rightarrow \infty$)	
Vertical Magnetic Dipole Source					
$e_\phi = \frac{-m}{2\pi\sigma r^4} \left[3 \operatorname{erf}(\theta r) - \frac{2}{\sqrt{\pi}} \theta r (3 + 2\theta^2 r^2) e^{-\theta^2 r^2} \right]$ (loop-wire)	$e_\phi^0 = 0$	$e_\phi^e = \frac{-3m}{2\pi\sigma r^4}$	$\rho_a^e = \frac{-2\pi r^4}{3m} e_\phi$	$e_\phi^\ell = \frac{-m\sigma^{3/2}\mu_0^{5/2}r}{40\pi^{3/2}t^{5/2}}$	$\rho_a^\ell = \frac{m^{2/3}\mu_0^{5/3}r^{2/3}}{40^{2/3}\pi t^{5/3}} (-e_\phi)^{-2/3}$
$h_r = \frac{-m\theta^2}{2\pi r} e^{-\theta^2 r^2} \left[\left(1 + \frac{4}{\theta^2 r^2} \right) I_1 \left(\frac{\theta^2 r^2}{2} \right) - I_0 \left(\frac{\theta^2 r^2}{2} \right) \right]$ (perpendicular)	$h_r^0 = 0$	$h_r^e = \frac{-3m t^{1/2}}{\pi^{3/2}\sigma^{1/2}\mu_0^{1/2}r^4}$	$\rho_a^e = \frac{\pi^3\mu_0 r^8}{9m^2 t} h_r^2$	$h_r^\ell = \frac{-m\sigma^2\mu_0^2 r}{128\pi t^2}$	$\rho_a^\ell = \frac{m^{1/2}\mu_0 r^{1/2}}{8(2\pi)^{1/2}t} (-h_r)^{-1/2}$
$\frac{\partial h_r}{\partial t} = \frac{m\theta^2}{2\pi r t} e^{-\theta^2 r^2} \left[(1 + \theta^2 r^2) I_0 \left(\frac{\theta^2 r^2}{2} \right) - \left(2 + \theta^2 r^2 + \frac{4}{\theta^2 r^2} \right) I_1 \left(\frac{\theta^2 r^2}{2} \right) \right]$		$\frac{\partial h_r^e}{\partial t} = \frac{-3m}{2\pi^{3/2}\sigma^{1/2}\mu_0^{1/2}r^4 t^{1/2}}$	$\rho_a^e = \frac{4\pi^3\mu_0 r^8 t}{9m^2} \left(\frac{\partial h_r}{\partial t} \right)^2$	$\frac{\partial h_r^\ell}{\partial t} = \frac{m\mu_0^2\sigma^2 r}{64\pi t^3}$	$\rho_a^\ell = \frac{m^{1/2}\mu_0 r^{1/2}}{8\pi^{1/2}t^{3/2}} \left(\frac{\partial h_r}{\partial t} \right)^{-1/2}$
$h_z = \frac{m}{4\pi r^3} \left[\frac{9}{2\theta^2 r^2} \operatorname{erf}(\theta r) - \operatorname{erf}(\theta r) - \frac{1}{\sqrt{\pi}} \left(\frac{9}{\theta r} + 4\theta r \right) e^{-\theta^2 r^2} \right]$ (horizontal coplanar)	$h_z^0 = \frac{-m}{4\pi r^3}$	$h_z^e = \frac{-m}{4\pi r^3} \left(1 - \frac{18t}{\sigma\mu_0 r^2} \right)$	$\rho_a^e = \frac{2\pi\mu_0 r^5}{9m t} \left(\frac{m}{4\pi r^3} + h_z \right)$	$h_z^\ell = \frac{m\sigma^{3/2}\mu_0^{3/2}}{30\pi^{3/2}t^{3/2}}$	$\rho_a^\ell = \frac{m^{2/3}\mu_0}{30^{2/3}\pi t} (h_z)^{-2/3}$
$\frac{\partial h_z}{\partial t} = \frac{m}{2\pi\sigma\mu_0 r^5} \left[9 \operatorname{erf}(\theta r) - \frac{2\theta r}{\sqrt{\pi}} (9 + 6\theta^2 r^2 + 4\theta^4 r^4) e^{-\theta^2 r^2} \right]$		$\frac{\partial h_z^e}{\partial t} = \frac{9m}{2\pi\sigma\mu_0 r^5}$	$\rho_a^e = \frac{2\pi\mu_0 r^5}{9m} \frac{\partial h_z}{\partial t}$	$\frac{\partial h_z^\ell}{\partial t} = \frac{-m\sigma^{3/2}\mu_0^{3/2}}{20\pi^{3/2}t^{5/2}}$	$\rho_a^\ell = \frac{m^{2/3}\mu_0}{20^{2/3}\pi t^{5/3}} \left(\frac{-\partial h_z}{\partial t} \right)^{-2/3}$
$\theta = \left(\frac{\sigma\mu_0}{4t} \right)^{1/2}$					

Métodos eletromagnéticos

Table 2. Continued

General Expression	D.C. Value ($t < 0$)	Early-Time Asymptote ($\theta r \gg 1, t \rightarrow 0$)		Late-Time Asymptote ($\theta r \ll 1, t \rightarrow \infty$)	
Horizontal Electric Dipole (x-directed)					
$e_x = e_r = \frac{p}{2\pi\sigma r^3} \left[\operatorname{erf}(\theta r) - \frac{2}{\sqrt{\pi}} \theta r e^{-\theta^2 r^2} \right]$ (in-line array)	$e_x^0 = \frac{p}{\pi\sigma r^3}$	$e_x^e = \frac{p}{2\pi\sigma r^3}$	$\rho_a^e = \frac{2\pi r^3}{p} e_x$	$e_x^\ell = \frac{p\sigma^{1/2}\mu_0^{3/2}}{12\pi^{3/2}t^{3/2}}$	$\rho_a^\ell = \frac{p^2\mu_0^3}{144\pi^3 t^3} (e_x)^{-2}$
$e_x = -e_\phi = e_r$ (in-line) $= \frac{p}{2\pi\sigma r^3} \left[\operatorname{erf}(\theta r) - \frac{2}{\sqrt{\pi}} \theta r e^{-\theta^2 r^2} \right]$ (equatorial array)	$e_x^0 = \frac{-p}{2\pi\sigma r^3}$	$e_x^e = \frac{p}{2\pi\sigma r^3}$	$\rho_a^e = \frac{2\pi r^3}{p} e_x$	$e_x^\ell = \frac{p\sigma^{1/2}\mu_0^{3/2}}{12\pi^{3/2}t^{3/2}}$	$\rho_a^\ell = \frac{p^2\mu_0^3}{144\pi^3 t^3} (e_x)^{-2}$
$h_z = \frac{pt}{2\pi\sigma\mu_0 r^4} \left[(2\theta^2 r^2 - 3) \operatorname{erf}(\theta r) + \frac{6}{\sqrt{\pi}} \theta r e^{-\theta^2 r^2} \right]$	$h_z^0 = \frac{p}{4\pi r^2}$	$h_z^e = \frac{p}{4\pi r^2} \left(1 - \frac{6t}{\sigma\mu_0 r^2} \right)$	$\rho_a^e = \frac{2\pi\mu_0 r^4}{3pt} \left(\frac{p}{4\pi r^2} - h_z \right)$	$h_z^\ell = \frac{p\sigma^{3/2}\mu_0^{3/2}r}{60\pi^{3/2}t^{3/2}}$	$\rho_a^\ell = \frac{p^{2/3}\mu_0 r^{2/3}}{60^{2/3}\pi t} (h_z)^{-2/3}$
$\frac{\partial h_z}{\partial t} = \frac{p}{2\pi\sigma\mu_0 r^4} \left[3 \operatorname{erf}(\theta r) - \frac{2}{\sqrt{\pi}} \theta r (3 + 2\theta^2 r^2) e^{-\theta^2 r^2} \right]$		$\frac{\partial h_z^e}{\partial t} = \frac{3p}{2\pi\sigma\mu_0 r^4}$	$\rho_a^e = \frac{2\pi\mu_0 r^4}{3p} \frac{\partial h_z}{\partial t}$	$\frac{\partial h_z^\ell}{\partial t} = \frac{p\sigma^{3/2}\mu_0^{3/2}r}{40\pi^{3/2}t^{5/2}}$	$\rho_a^\ell = \frac{p^{2/3}\mu_0 r^{2/3}}{40^{2/3}\pi t^{5/3}} \left(\frac{\partial h_z}{\partial t} \right)^{-2/3}$
$h_r = \frac{p}{4\pi r^3} \left\{ \left[I_0 \left(\frac{\theta^2 r^2}{2} \right) + 3I_1 \left(\frac{\theta^2 r^2}{2} \right) \right] e^{-\frac{\theta^2 r^2}{2}} - 1 \right\}$	$h_r^0 = \frac{-p}{4\pi r^2}$	$h_r^e = \frac{-p}{4\pi r^2} \left[1 - \frac{8}{r} \left(\frac{t}{\pi\sigma\mu_0} \right)^{1/2} \right]$	$\rho_a^e = \frac{\pi\mu_0 r^2}{64t} \left(1 + \frac{4\pi r^2}{p} h_r \right)^2$	$h_r^\ell = \frac{p\sigma\mu_0}{64\pi t}$	$\rho_a^\ell = \frac{p\mu_0}{64\pi t} (h_r)^{-1}$
$\frac{\partial h_r}{\partial t} = \frac{-p}{4\pi r^3 t} \left[\theta^2 r^2 I_0 \left(\frac{\theta^2 r^2}{2} \right) - (\theta^2 r^2 + 3) I_1 \left(\frac{\theta^2 r^2}{2} \right) \right] e^{-\frac{\theta^2 r^2}{2}}$		$\frac{\partial h_r^e}{\partial t} = \frac{p}{\pi r^3 (\pi\sigma\mu_0 t)^{1/2}}$	$\rho_a^e = \frac{\pi^3 \mu_0 r^6 t}{p^2} \left(\frac{\partial h_r}{\partial t} \right)^2$	$\frac{\partial h_r^\ell}{\partial t} = \frac{-p\sigma\mu_0}{64\pi t^2}$	$\rho_a^\ell = \frac{-p\mu_0}{64\pi t^2} \left(\frac{\partial h_r}{\partial t} \right)^{-1}$
$h_\phi = \frac{-p}{4\pi r^3} \left\{ \left[I_0 \left(\frac{\theta^2 r^2}{2} \right) + I_1 \left(\frac{\theta^2 r^2}{2} \right) \right] e^{-\frac{\theta^2 r^2}{2}} - 1 \right\}$ (in-line)	$h_\phi^0 = \frac{p}{4\pi r^2}$	$h_\phi^e = \frac{p}{4\pi r^2} \left[1 - \frac{4}{r} \left(\frac{t}{\pi\sigma\mu_0} \right)^{1/2} \right]$	$\rho_a^e = \frac{\pi^3 \mu_0 r^6}{p^2 t} \left(\frac{p}{4\pi r^2} - h_\phi \right)^2$	$h_\phi^\ell = \frac{p\sigma\mu_0}{64\pi t}$	$\rho_a^\ell = \frac{p\mu_0}{64\pi t} (h_\phi)^{-1}$
$\frac{\partial h_\phi}{\partial t} = \frac{-p}{4\pi r^3 t} I_1 \left(\frac{\theta^2 r^2}{2} \right) e^{-\frac{\theta^2 r^2}{2}}$		$\frac{\partial h_\phi^e}{\partial t} = \frac{-p}{2\pi r^3 (\pi\sigma\mu_0 t)^{1/2}}$	$\rho_a^e = \frac{4\pi^3 \mu_0 r^6 t}{p^2} \left(\frac{\partial h_\phi}{\partial t} \right)^2$	$\frac{\partial h_\phi^\ell}{\partial t} = \frac{-p\sigma\mu_0}{64\pi t^2}$	$\rho_a^\ell = \frac{-p\mu_0}{64\pi t^2} \left(\frac{\partial h_\phi}{\partial t} \right)^{-1}$

Spies and Frischknecht

Métodos eletromagnéticos

Table 2. Continued

General Expression	D.C. Value ($t < 0$)	Early-Time Asymptote ($\theta r \gg 1, t \rightarrow 0$)	Late-Time Asymptote ($\theta r \ll 1, t \rightarrow \infty$)
Large Horizontal Loop			
$h_z = \frac{It}{\sigma\mu_0 a^3} \left[(2\theta^2 a^2 - 3) \operatorname{erf}(\theta a) + \frac{6}{\sqrt{\pi}} \theta a e^{-\theta^2 a^2} \right]$ (central loop; in loop)	$h_z^0 = \frac{I}{2a}$	$h_z^e = \frac{I}{2a} \left[1 - \frac{6t}{\sigma\mu_0 a^2} \right] \quad \rho_a^e = \frac{\mu_0 a^3}{3It} \left(\frac{I}{2a} - h_z \right)$	$h_z^e = \frac{I\sigma^{3/2}\mu_0^{3/2}a^2}{30\pi^{1/2}t^{3/2}} \quad \rho_a^e = \frac{I^{2/3}\mu_0^{2/3}2^{4/3}}{30^{2/3}\pi^{1/3}t^{5/3}} (h_z)^{-2/3}$
$\frac{\partial h_z}{\partial t} = \frac{-I}{\sigma\mu_0 a^3} \left[3 \operatorname{erf}(\theta a) - \frac{2}{\sqrt{\pi}} \theta a (3 + 2\theta^2 a^2) e^{-\theta^2 a^2} \right]$		$\frac{\partial h_z^e}{\partial t} = \frac{-3I}{\sigma\mu_0 a^3} \quad \rho_a^e = \frac{-\mu_0 a^3}{3I} \frac{\partial h_z}{\partial t}$	$\frac{\partial h_z^e}{\partial t} = \frac{-I\sigma^{3/2}\mu_0^{3/2}a^2}{20\pi^{1/2}t^{5/2}} \quad \rho_a^e = \frac{I^{2/3}\mu_0^{4/3}}{20^{2/3}\pi^{1/3}t^{5/3}} \left(\frac{-\partial h_z}{\partial t} \right)^{-2/3}$
Single Loop (coincident Loop)			
$v = \frac{-2Ia\mu_0\sqrt{\pi}}{t} (\theta a)^3 \sum_{m=0}^{\infty} \frac{(-1)^m (2m+2)! (\theta a)^{2m}}{m!(m+1)!(m+2)!(2m+5)}$	$v^0 = 0$	$v^e = \frac{-I\mu_0 a}{2t} \quad \rho_a^e \text{ is undefined}$	$v^e = \frac{-\sqrt{\pi}I\sigma^{3/2}\mu_0^{5/2}a^4}{20t^{5/2}} \quad \rho_a^e = \frac{I^{2/3}\pi^{1/3}\mu_0^{5/3}a^{8/3}}{20^{2/3}t^{5/3}} (-v)^{-2/3}$
Line source (x-directed)			
$e_x = \frac{I}{\pi\sigma y^2} (1 - e^{-\theta^2 y^2})$	$e_x^0 = 0$	$e_x^e = \frac{I}{\pi\sigma y^2} \quad \rho_a^e = \frac{\pi y^2}{I} e_x$	$e_x^e = \frac{\mu_0 I}{4\pi t} \left(1 - \frac{\sigma\mu_0 y^2}{8t} \right) \quad \rho_a^e = \frac{I\mu_0^2 y^2}{8t(\mu_0 I - 4\pi t e_x)}$
$\frac{\partial e_x}{\partial t} = \frac{-\mu_0 I}{4\pi t^2} e^{-\theta^2 y^2}$		$\frac{\partial e_x^e}{\partial t} = 0 \quad \rho_a^e \text{ is undefined}$	$\frac{\partial e_x^e}{\partial t} = \frac{-\mu_0 I}{4\pi t^2} \quad \rho_a^e \text{ is undefined}$
$h_z = \frac{-I}{2\pi y} \left[1 + \frac{1}{\theta^2 y^2} (e^{-\theta^2 y^2} - 1) \right]$	$h_z^0 = \frac{-I}{2\pi y}$	$h_z^e = \frac{-I}{2\pi y} \left(1 - \frac{4t}{\sigma\mu_0 y^2} \right) \quad \rho_a^e = \frac{\pi\mu_0 y^3}{2It} \left(\frac{I}{2\pi y} + h_z \right)$	$h_z^e = \frac{-I\sigma\mu_0 y}{16\pi t} \quad \rho_a^e = \frac{I\mu_0 y}{16\pi t} (-h_z)^{-1}$
$\frac{\partial h_z}{\partial t} = \frac{2I}{\pi\sigma\mu_0 y^3} [1 - (1 + \theta^2 y^2) e^{-\theta^2 y^2}]$		$\frac{\partial h_z^e}{\partial t} = \frac{2I}{\pi\sigma\mu_0 r^3} \quad \rho_a^e = \frac{\pi\mu_0 r^3}{2I} \frac{\partial h_z}{\partial t}$	$\frac{\partial h_z^e}{\partial t} = \frac{I\sigma\mu_0 y}{16\pi t^2} \quad \rho_a^e = \frac{I\mu_0 y}{16\pi t^2} \left(\frac{\partial h_z}{\partial t} \right)^{-1}$
$h_y = \frac{I}{\pi^{3/2} y} \left[\frac{F(\theta y)}{\theta^2 y^2} - \frac{1}{\theta y} \right]$	$h_y^0 = 0$	$h_y^e = \frac{-2It^{1/2}}{\pi^{3/2}\sigma^{1/2}\mu_0^{1/2}y^2} \quad \rho_a^e = \frac{\pi^3\mu_0 y^4}{4I^2} h_y^2$	$h_y^e = \frac{-I\sigma^{1/2}\mu_0^{1/2}}{3\pi^{3/2}t^{1/2}} \quad \rho_a^e = \frac{I^2\mu_0}{9\pi^3 t} (h_y)^{-2}$
$\frac{\partial h_y}{\partial t} = \frac{I}{\pi^{3/2} y t} \left[\left(1 + \frac{1}{\theta^2 y^2} \right) F(\theta y) - \frac{1}{\theta y} \right]$		$\frac{\partial h_y^e}{\partial t} = \frac{I}{\pi^{3/2}\sigma^{1/2}\mu_0^{1/2}y^2 t^{1/2}} \quad \rho_a^e = \frac{\pi^3\mu_0 y^4 t}{I^2} \left(\frac{-\partial h_y}{\partial t} \right)^2$	$\frac{\partial h_y^e}{\partial t} = \frac{I\sigma^{1/2}\mu_0^{1/2}}{6\pi^{3/2}t^{3/2}} \quad \rho_a^e = \frac{I^2\mu_0}{36\pi^3 t^3} \left(\frac{\partial h_y}{\partial t} \right)^{-2}$