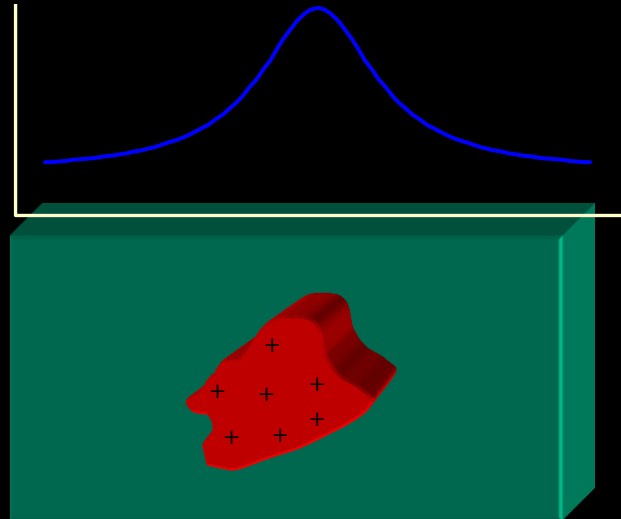


Delimitação de um corpo em profundidade

Convexidade




Problema geológico: localizar e delinear um corpo anômalo em profundidade a partir de um perfil gravimétrico ou magnético

Simplificações: corpo bidimensional

Informação a priori: corpo convexo, com contraste de densidade constante e conhecido

Estabilização


$$= C_0 \cdot f_0 + C_1 \cdot f_1 + C_2 \cdot f_2 + C_3 \cdot f_3 + \dots$$

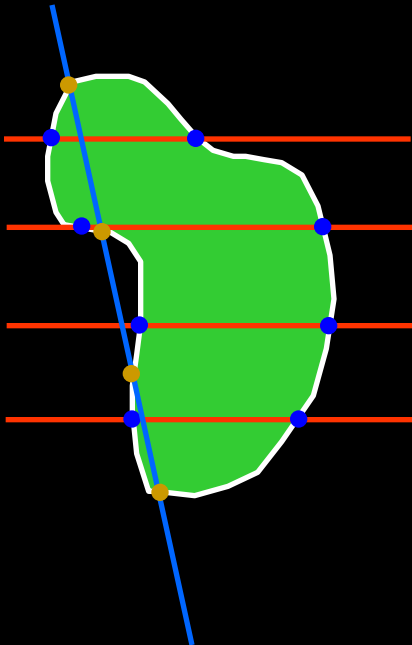
The equation illustrates the decomposition of a complex shape (represented by the yellow curve) into a series of simpler basis functions ($f_0, f_1, f_2, f_3, \dots$) weighted by coefficients ($C_0, C_1, C_2, C_3, \dots$). The basis functions are shown as red outlines: f_0 is a circle, f_1 is an ellipse, f_2 is a horizontally elongated shape with two indentations, and f_3 is a three-lobed shape. Ellipses are used to indicate that the series continues beyond the shown terms.

Convexidade

Direcional

Global

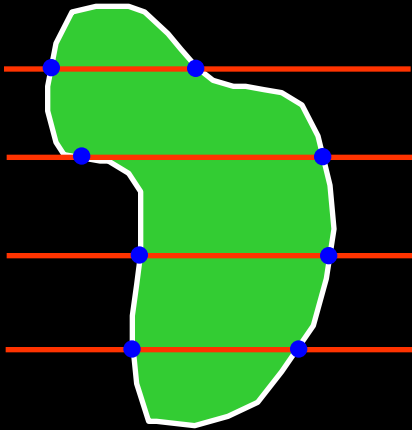
Retas ao longo de uma direção
cortam o corpo em apenas 2 pontos



Convexidade

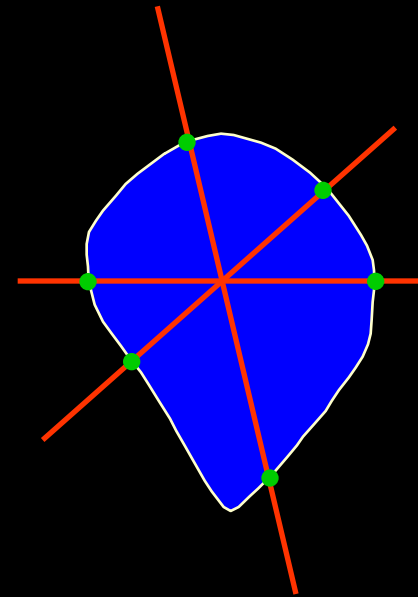
Direcional

Retas ao longo de uma direção cortam o corpo em apenas 2 pontos




Global

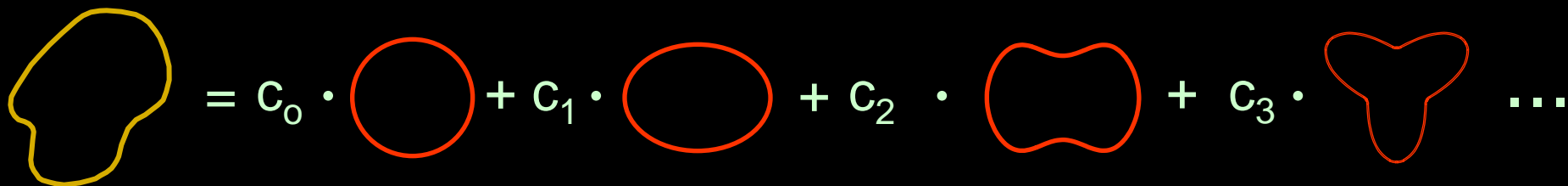
Retas ao longo de qualquer direção cortam o corpo em apenas 2 pontos



Estabilização


$$= C_0 \cdot \img alt="A red circle representing the first basis element." data-bbox="450 265 575 425"/> + C_1 \cdot \img alt="A red ellipse representing the second basis element." data-bbox="745 270 900 420"/> +$$
$$C_2 \cdot \img alt="A red figure-eight shape representing the third basis element." data-bbox="435 590 595 740"/> + C_3 \cdot \img alt="A red three-lobed shape representing the fourth basis element." data-bbox="745 575 900 755"/> \dots$$

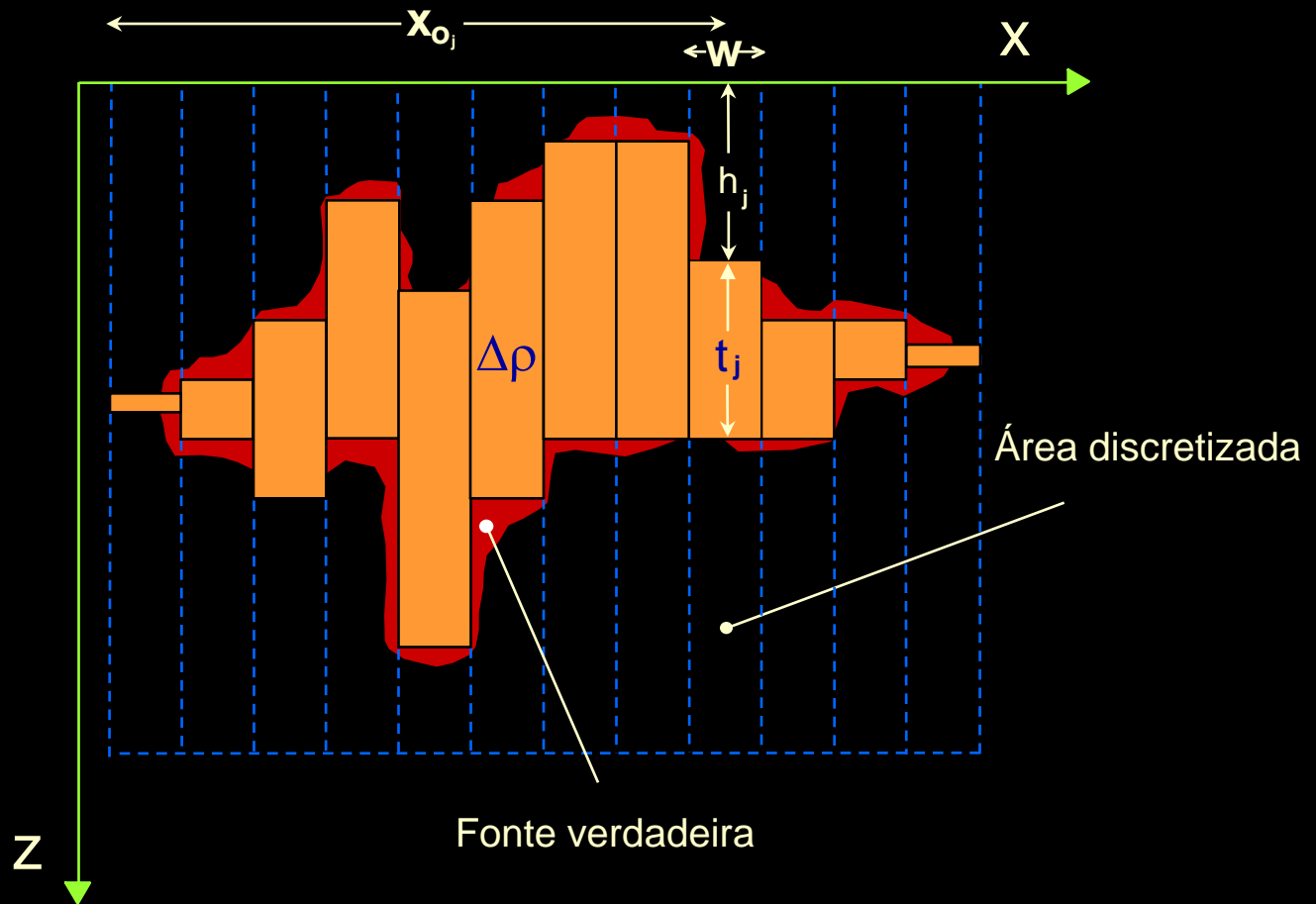
A convexidade global é mais estável


$$= c_0 \cdot \text{circle} + c_1 \cdot \text{ellipse} + c_2 \cdot \text{figure-eight} + c_3 \cdot \text{three-lobed} \dots$$

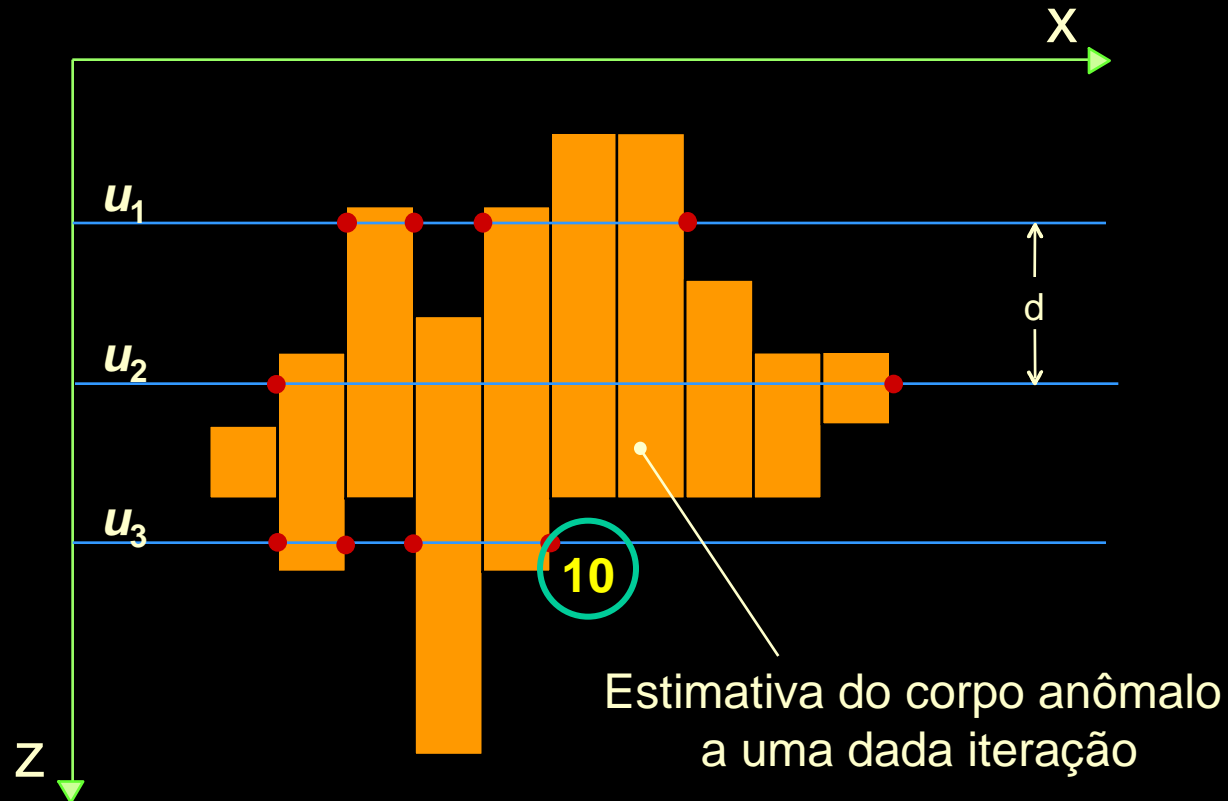
$$c_0 \gg c_i, \quad i = 2, 3 \dots \infty$$

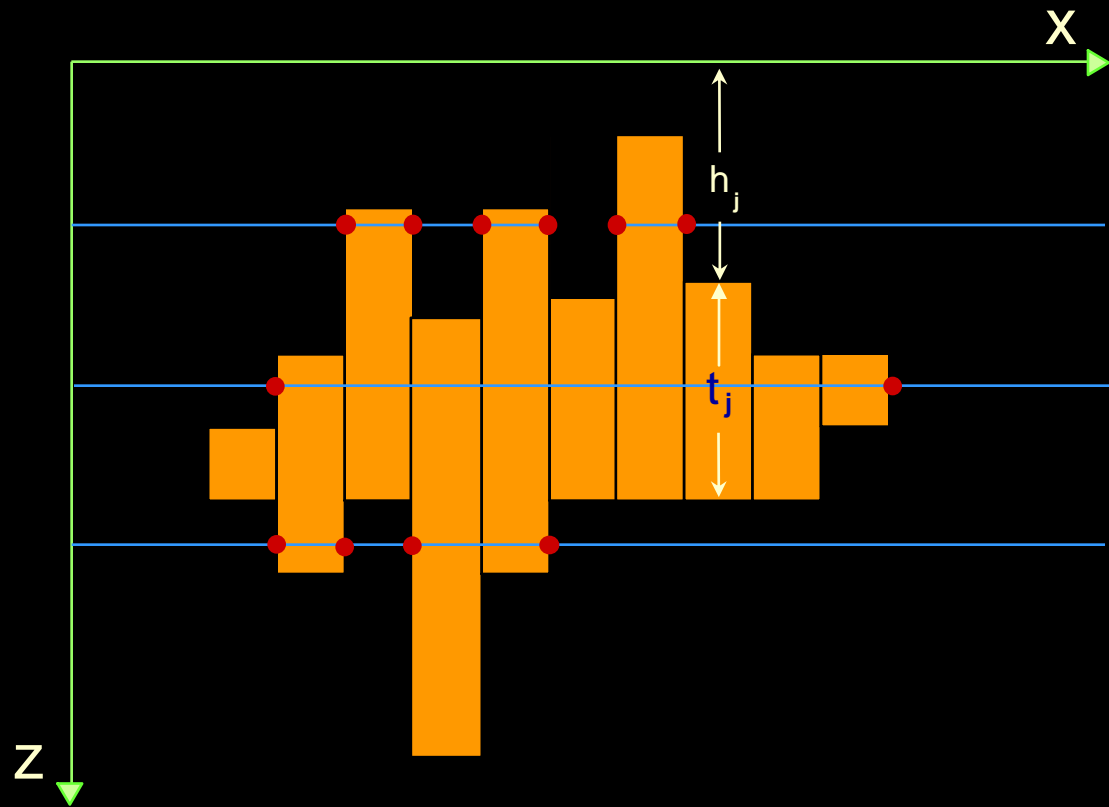
$$c_1 \gg c_i, \quad i = 2, 3 \dots \infty$$

Modelo interpretativo



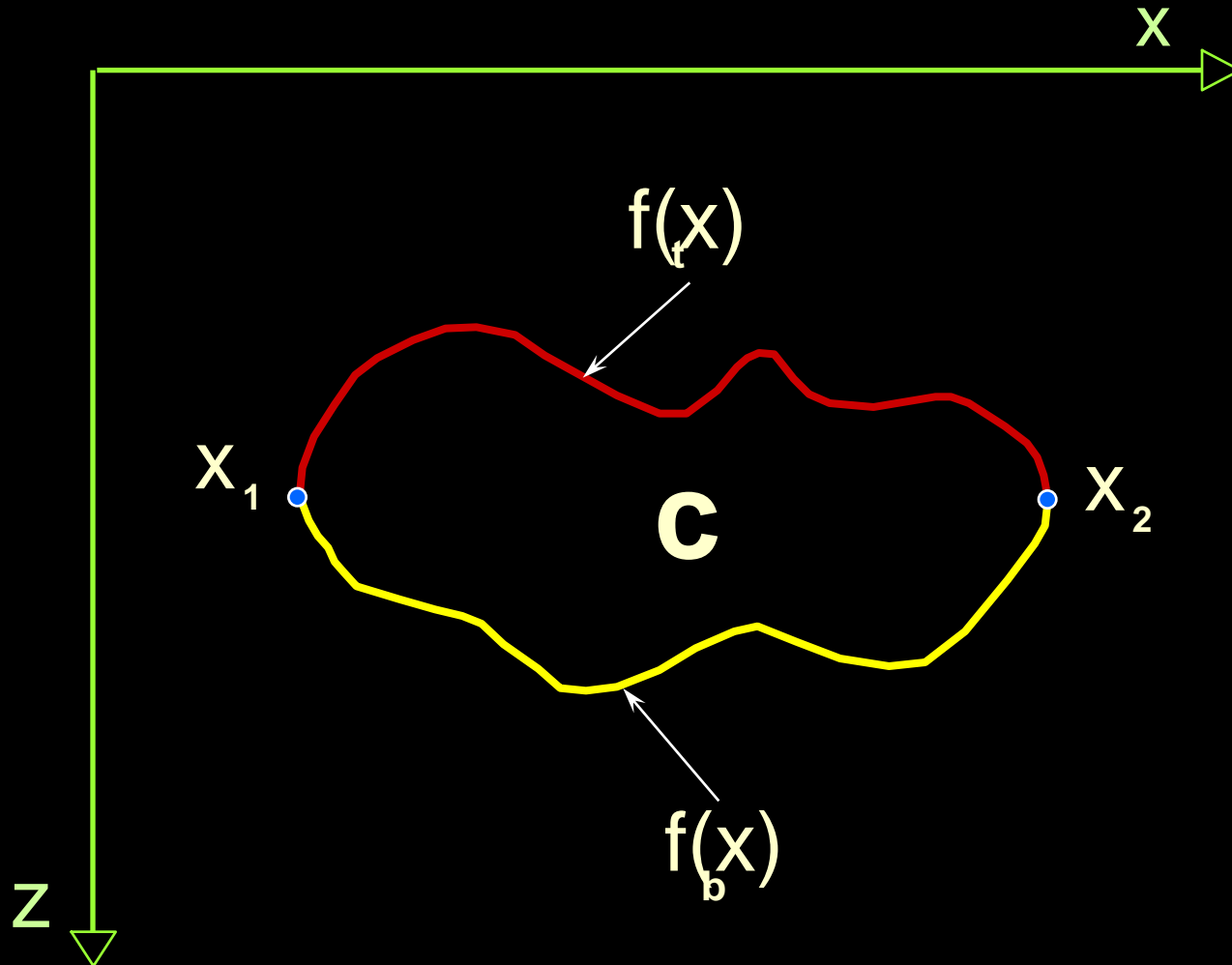
Convexidade direcional

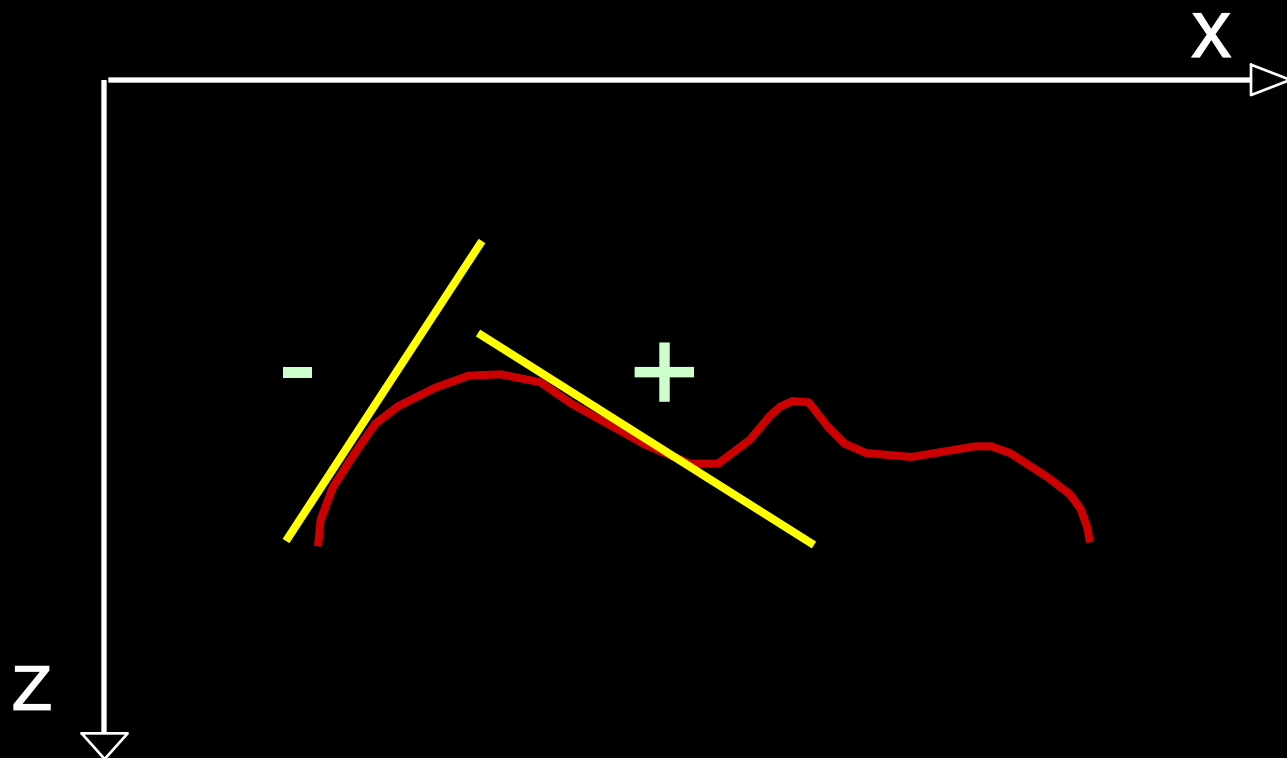




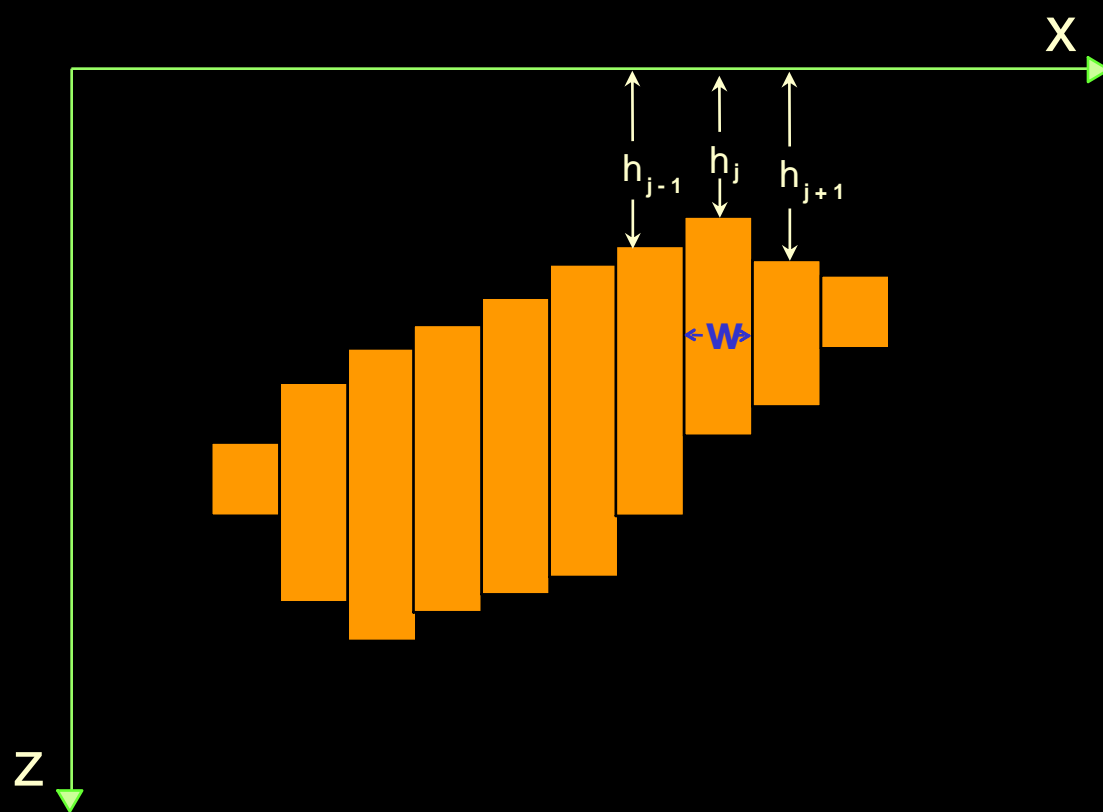
$$\left\{ \begin{array}{l} \min K(p) \\ \text{sujeito a} \\ [y^o - f(p)]^T [y^o - f(p)] = \delta \end{array} \right.$$

Convexidade global





$$\frac{\partial^2 f_t(x)}{\partial x^2} \geq 0$$



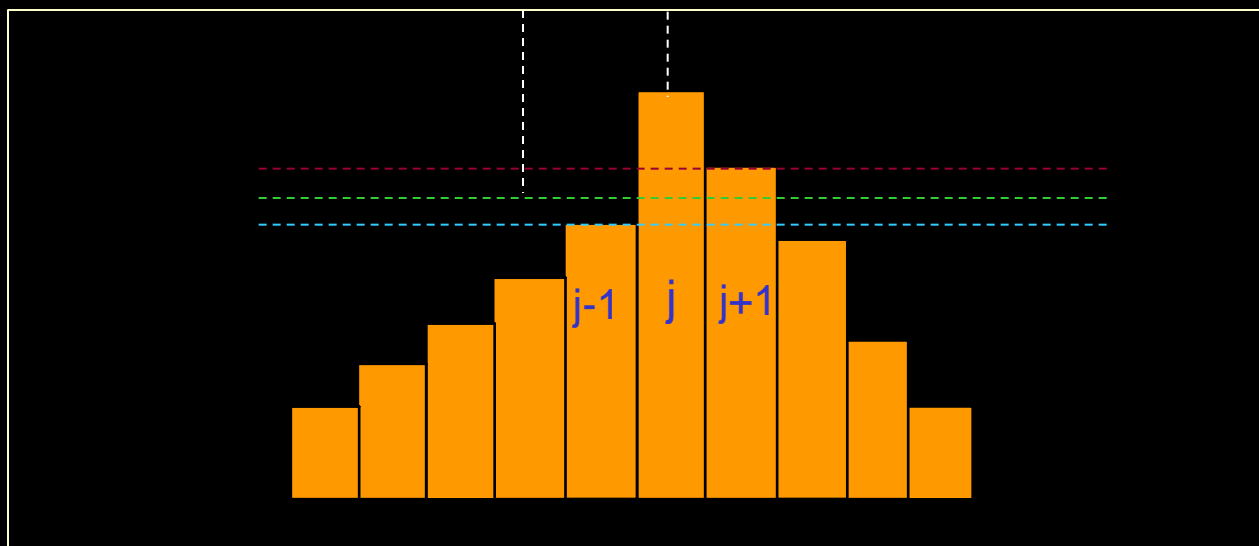
$$\frac{\partial f_t(x)}{\partial x} \approx \frac{h_{j+1} - h_j}{w}$$

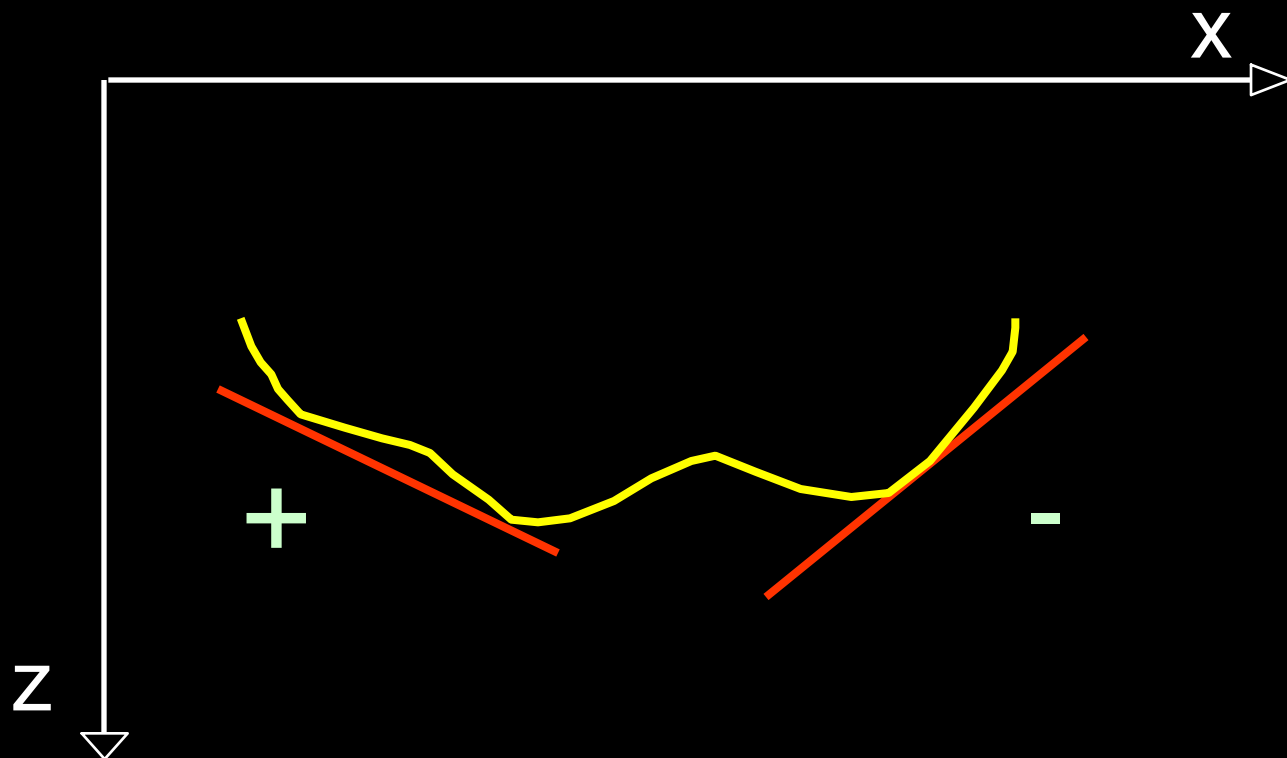
$$\frac{\partial f_t(x)}{\partial x} \approx \frac{h_j - h_{j-1}}{w}$$

$$\frac{\partial^2 f_t(x)}{\partial x^2} \approx \frac{h_{j+1} - 2h_j + h_{j-1}}{w^2}$$

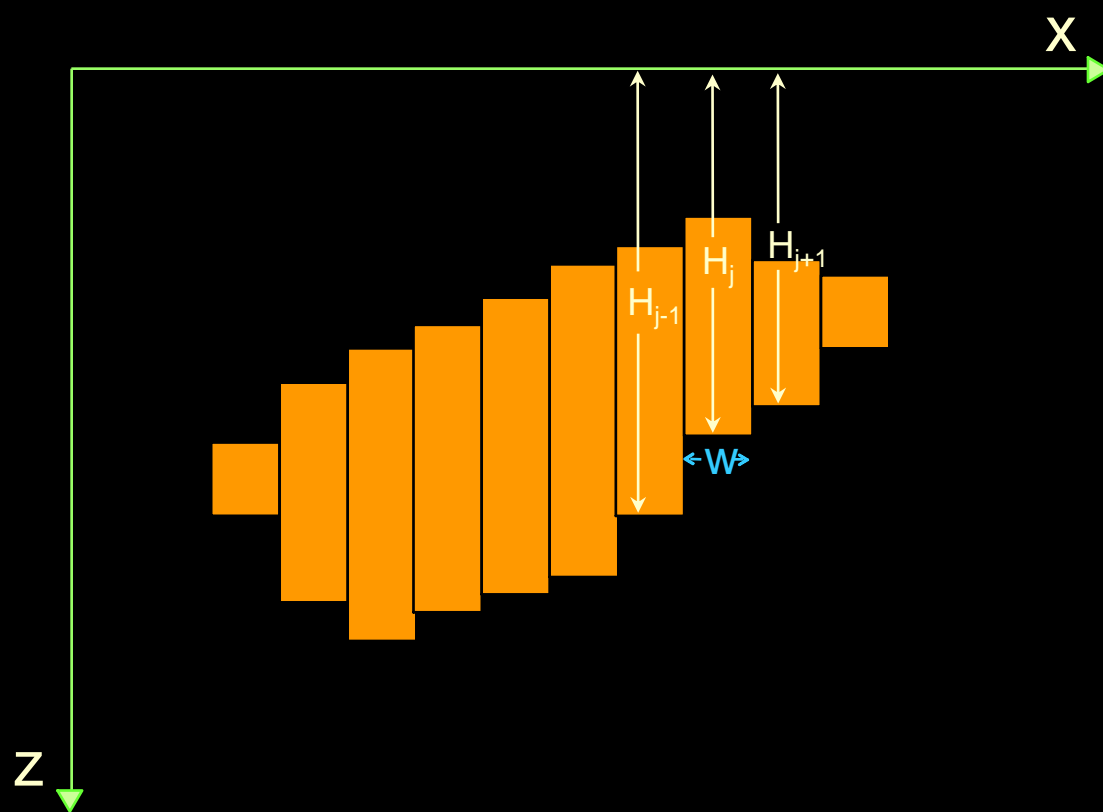
$$\frac{\partial^2 f_t(x)}{\partial x^2} \geq 0 \quad \longrightarrow \quad \frac{h_{j+1} - 2h_j + h_{j-1}}{w^2} \geq 0$$

$$h_j \leq \frac{h_{j-1} + h_{j+1}}{2}$$





$$\frac{\partial^2 f_b(x)}{\partial x^2} \leq 0$$



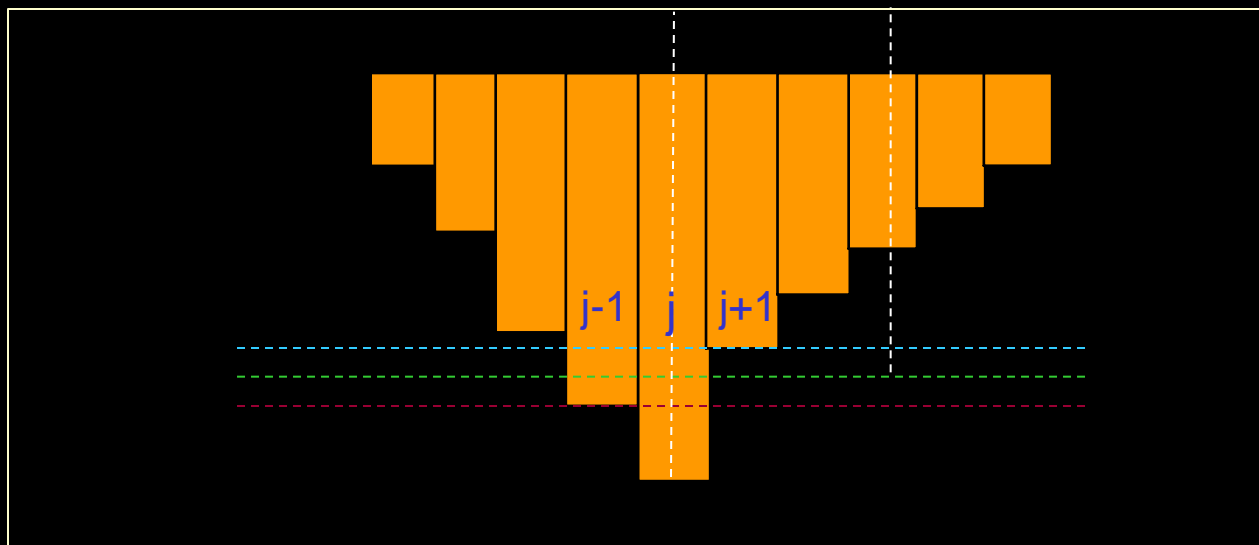
$$\frac{\partial f_b(x)}{\partial x} \approx \frac{H_{j+1} - H_j}{w}$$

$$\frac{\partial f_b(x)}{\partial x} \approx \frac{H_j - H_{j-1}}{w}$$

$$\frac{\partial^2 f_b(x)}{\partial x^2} \approx \frac{H_{j+1} - 2H_j + H_{j-1}}{w^2}$$

$$\frac{\partial^2 f_b(x)}{\partial x^2} \leq 0 \quad \longrightarrow \quad \frac{H_{j+1} - 2H_j + H_{j-1}}{w^2} \leq 0$$

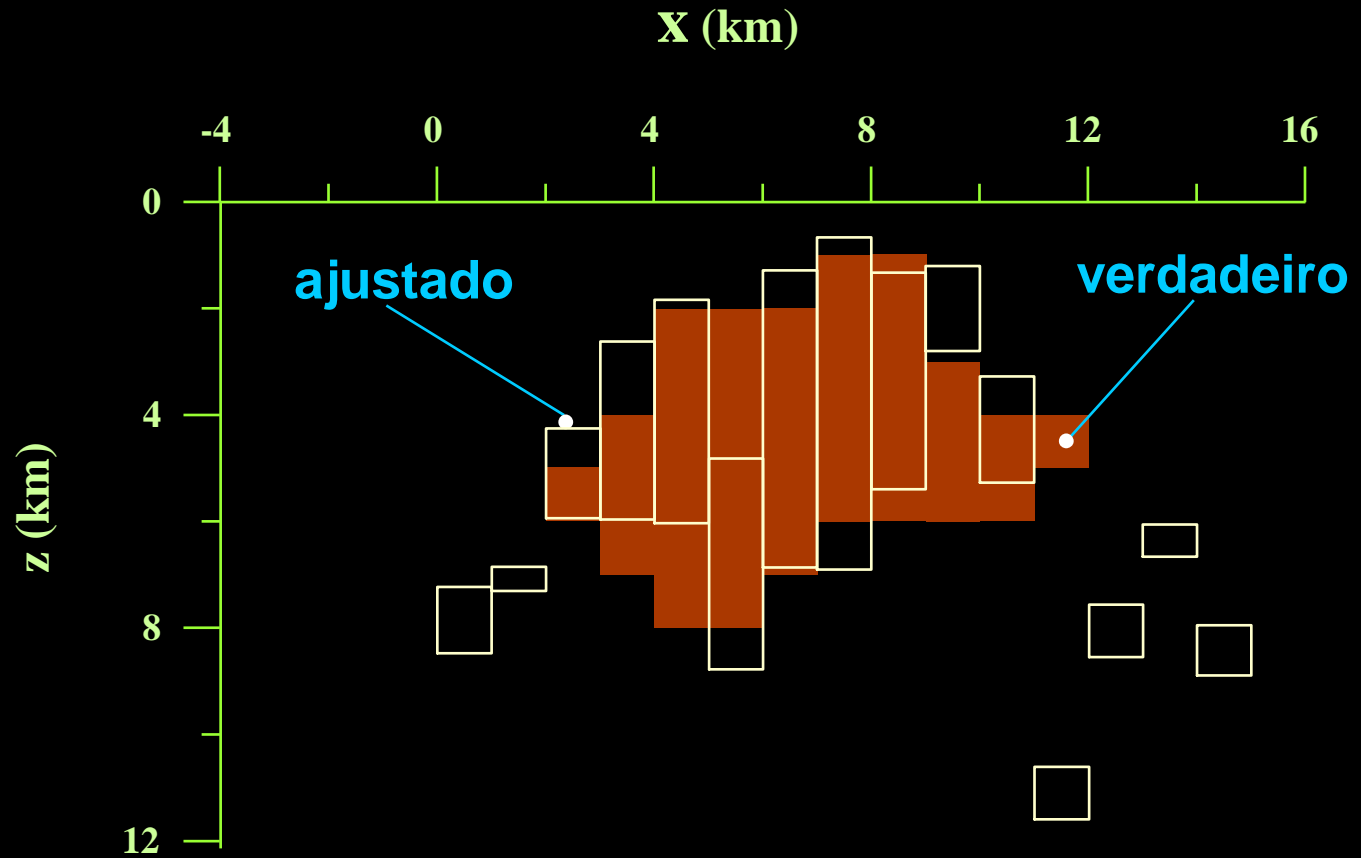
$$H_j \geq \frac{H_{j-1} + H_{j+1}}{2}$$



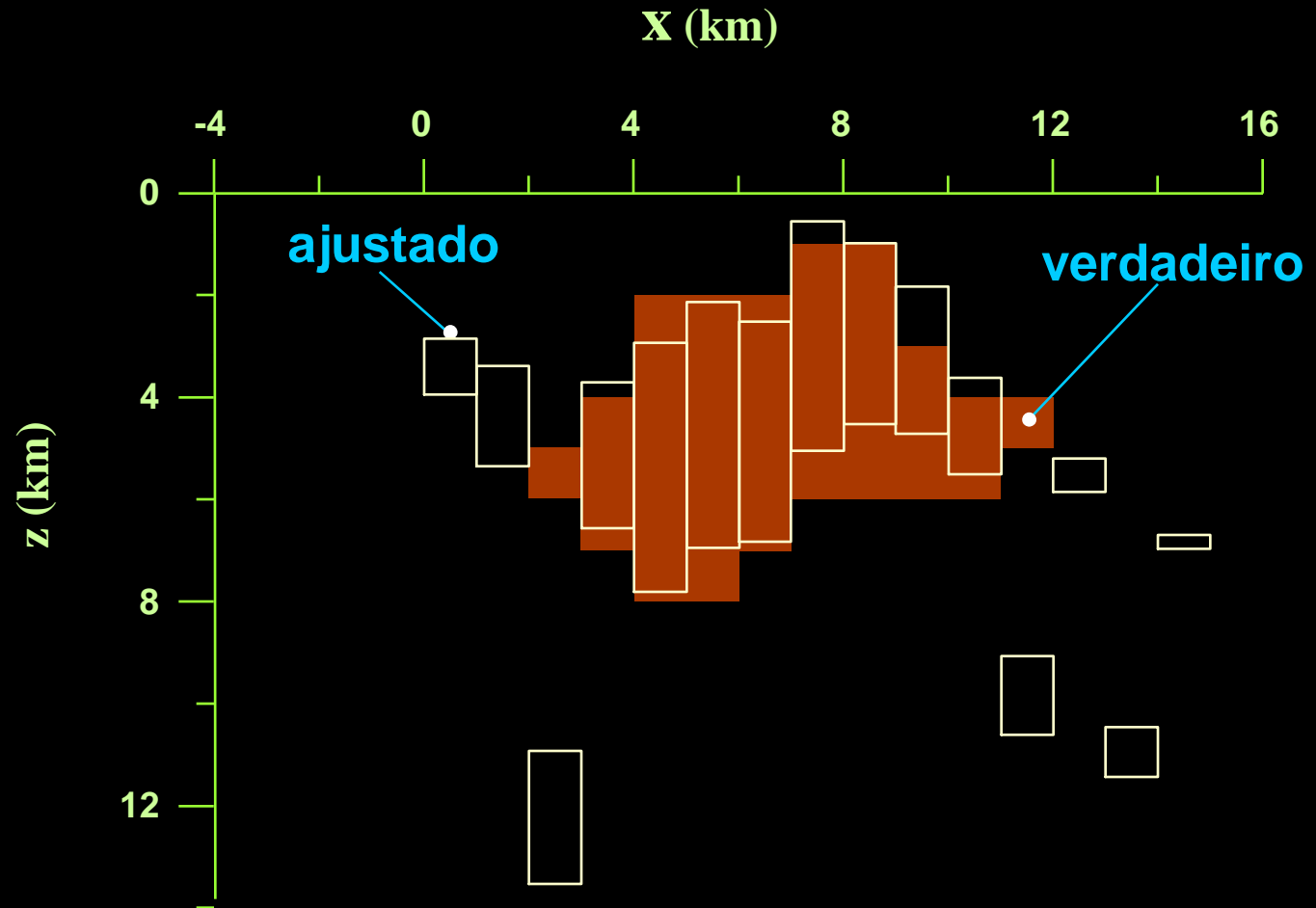
Convexidade global

$$\left\{ \begin{array}{l} \min [y^o - f(p)]^T [y^o - f(p)] = \delta \\ \text{sujeito a} \\ H_j \geq \frac{H_{j-1} + H_{j+1}}{2} \\ \text{e} \\ h_j \leq \frac{h_{j-1} + h_{j+1}}{2} \end{array} \right.$$

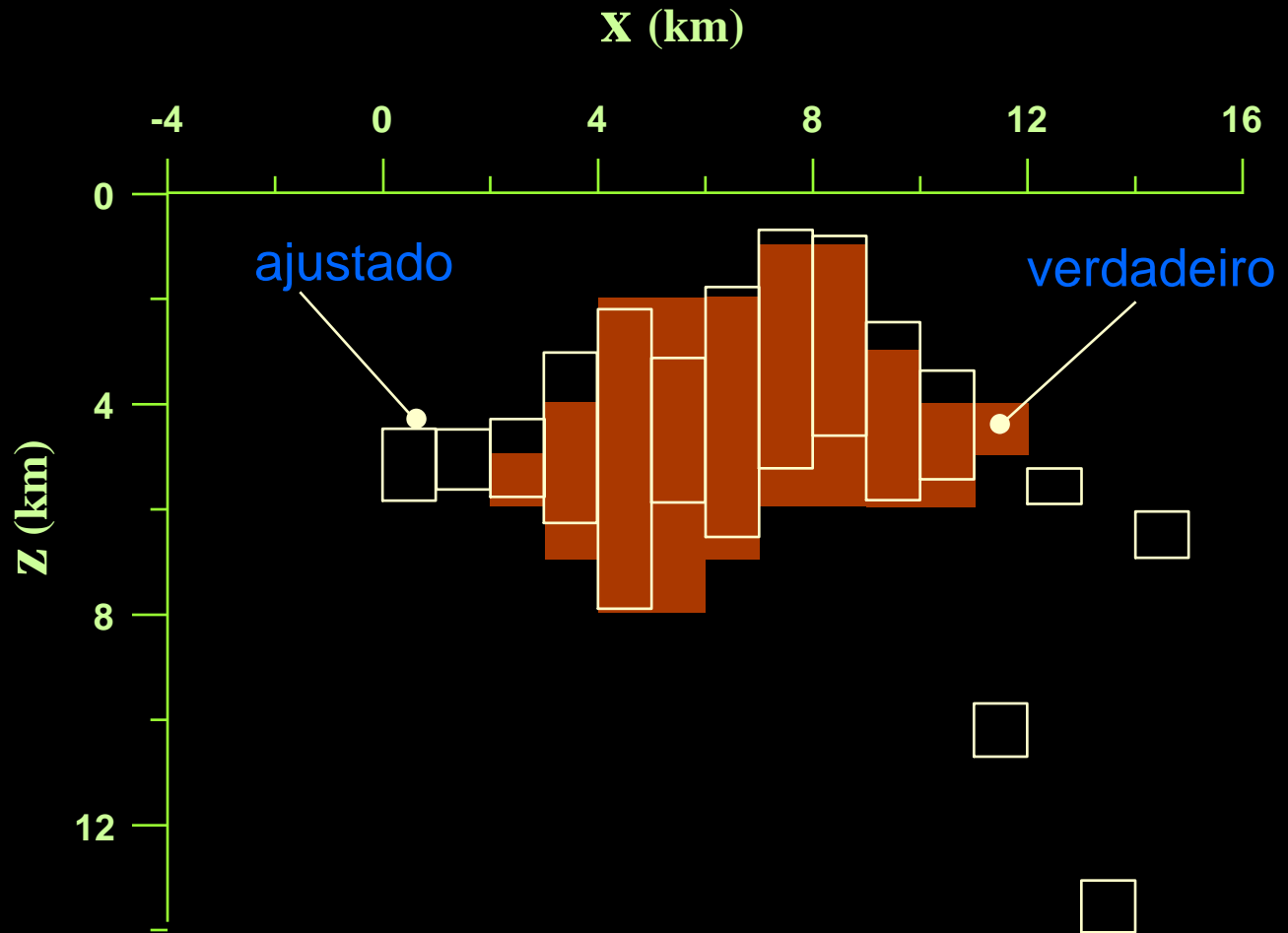
Batólito



Batólito

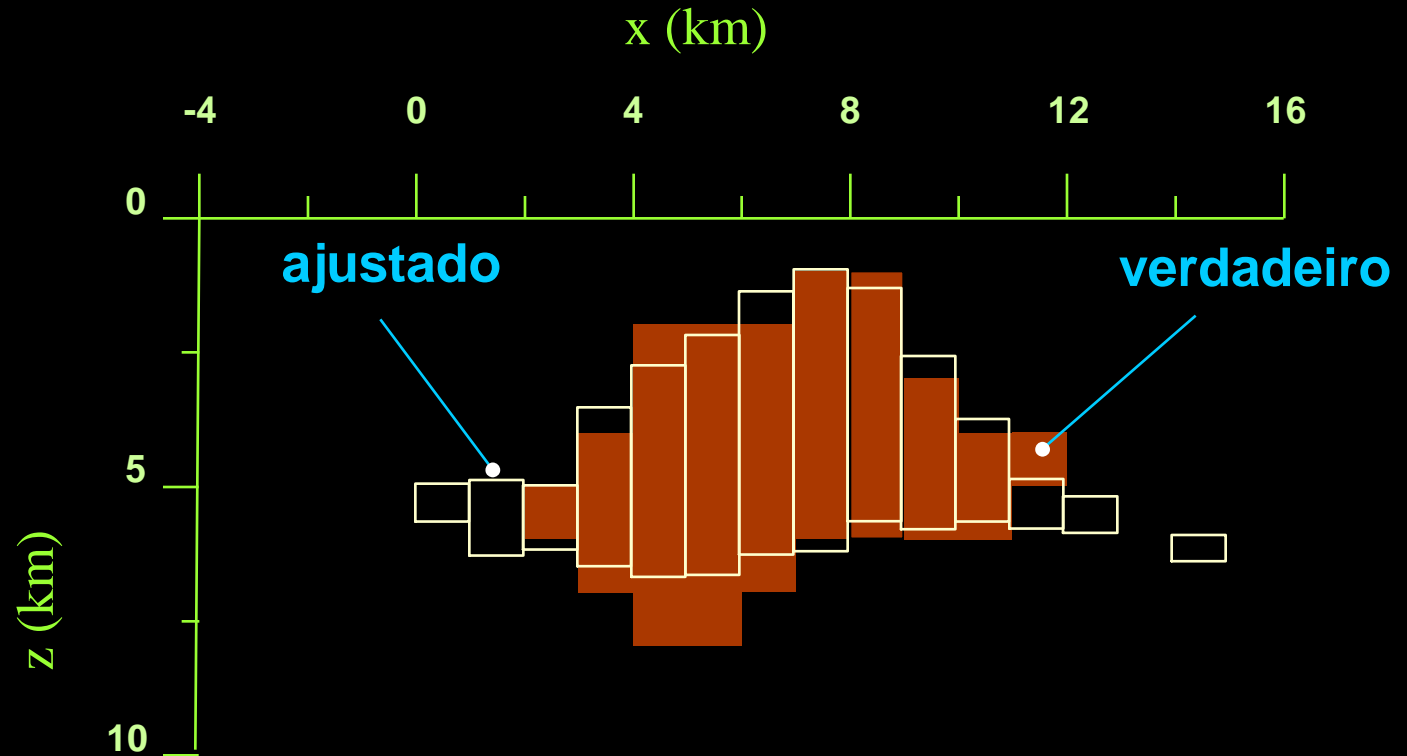


Batólito



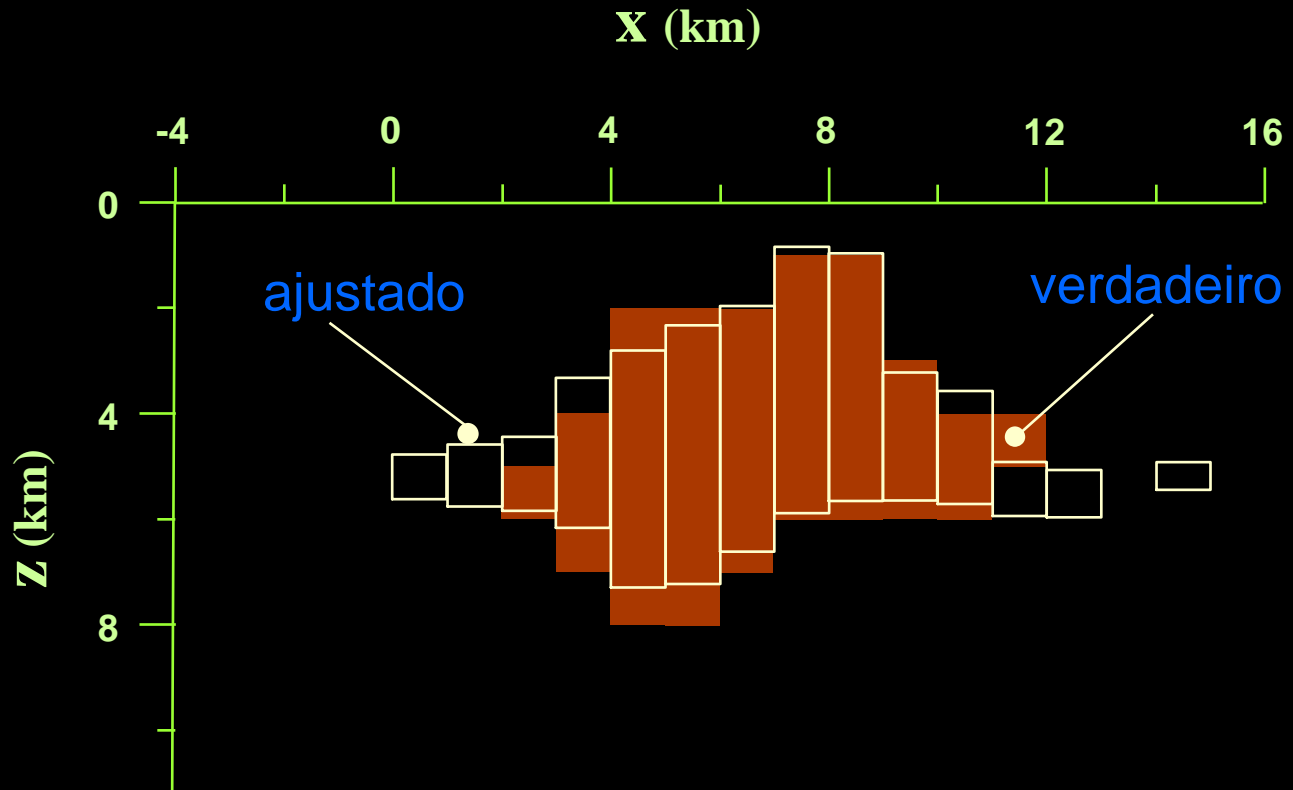
Batólito - convexidade em X

$d = 0,4 \text{ km}$



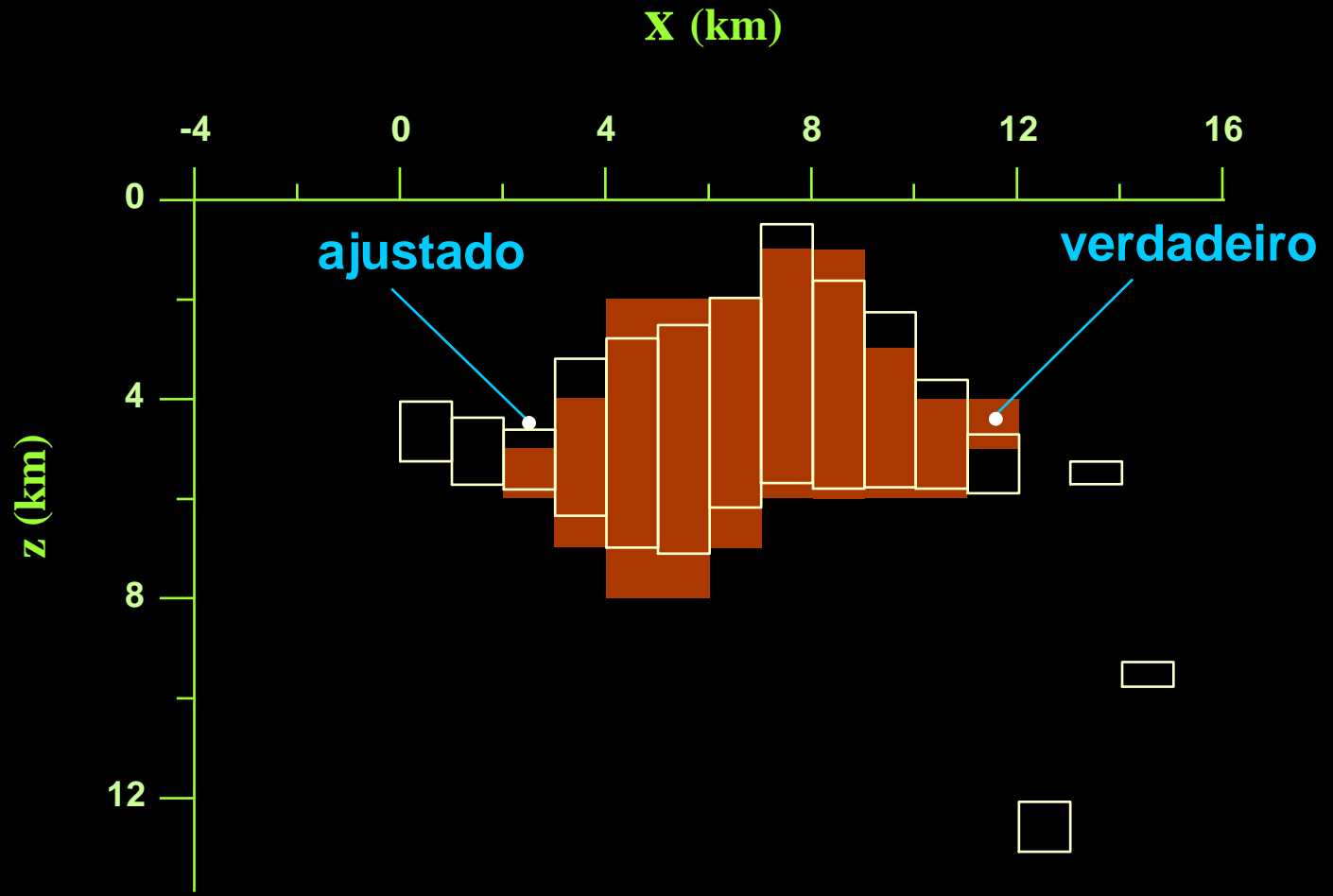
Batólito - convexidade em X

$d = 0,4 \text{ km}$



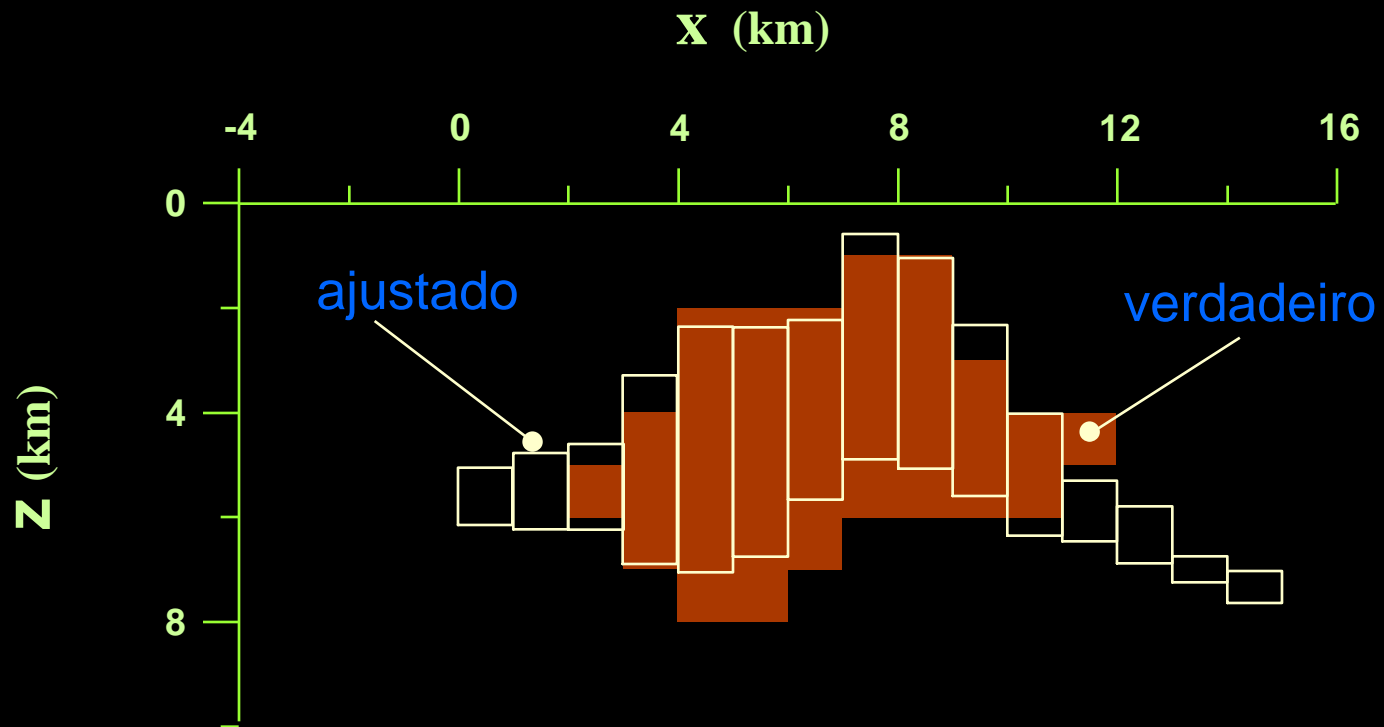
Batólito - convexidade em X

$d = 0,4 \text{ km}$



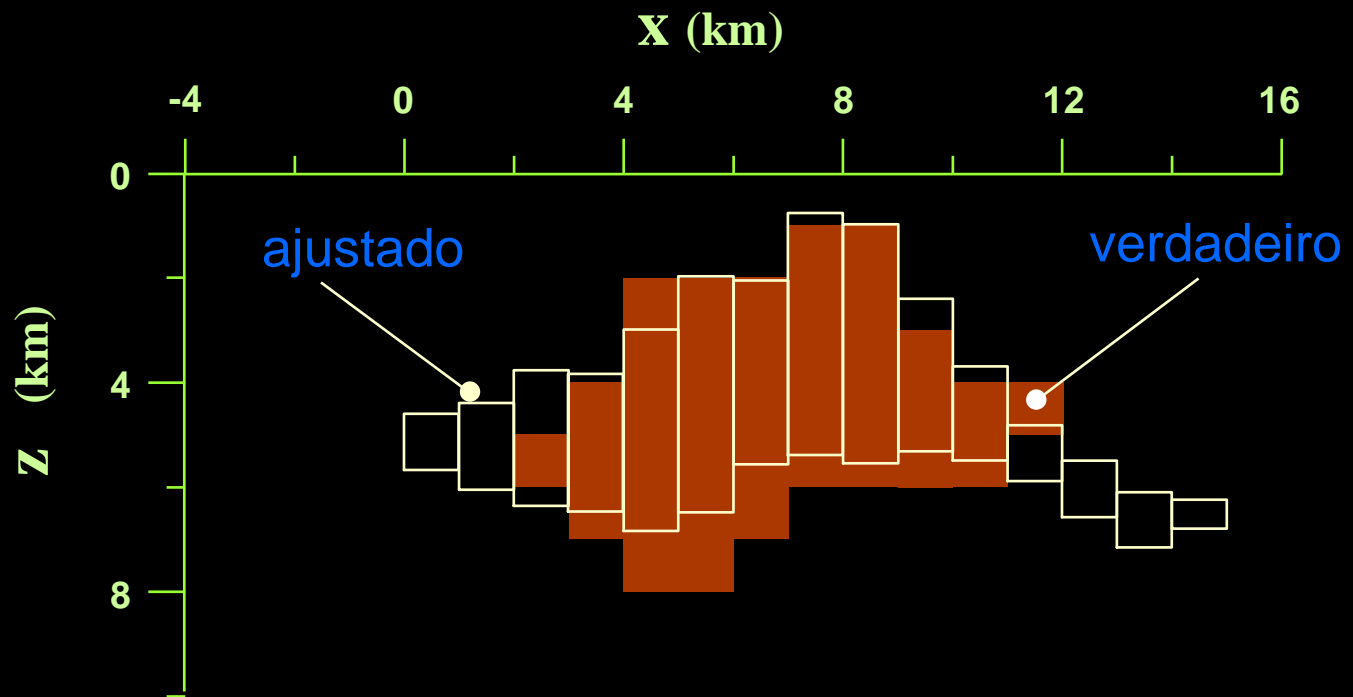
Batólito - suavidade

$\lambda = 0,25$



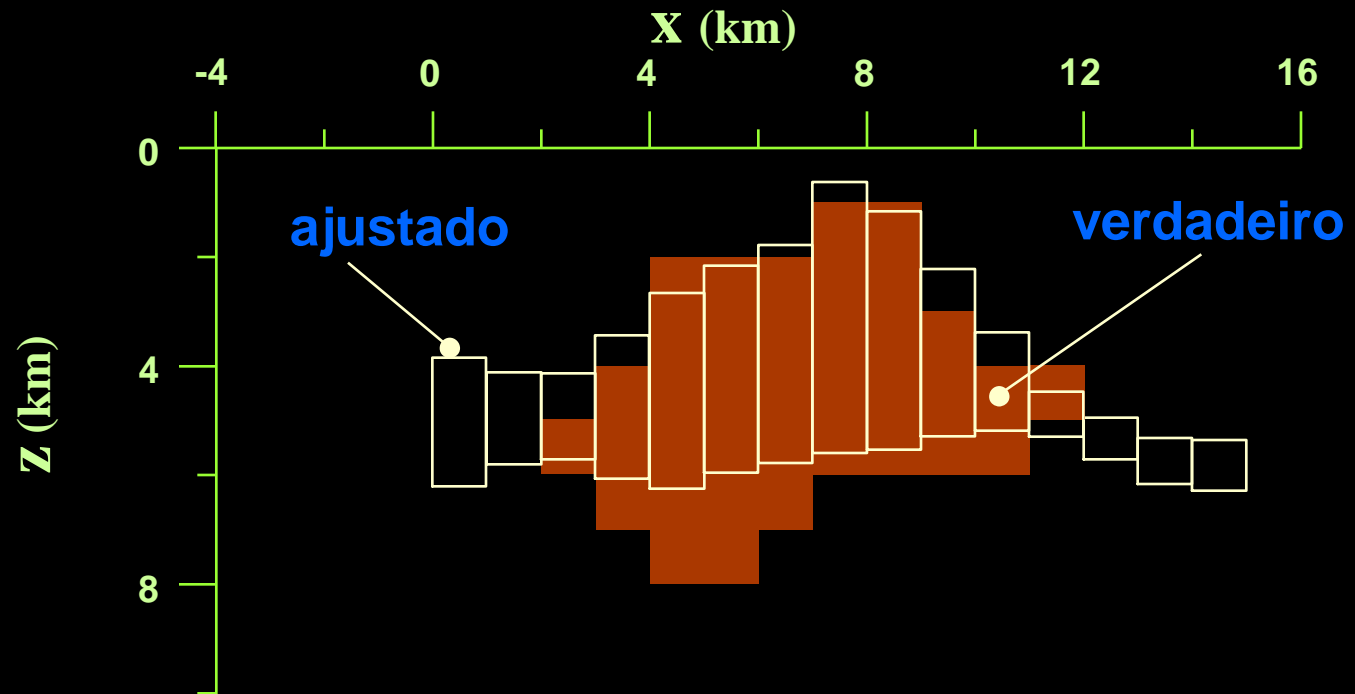
Batólito - suavidade

$$\lambda = 0,5$$

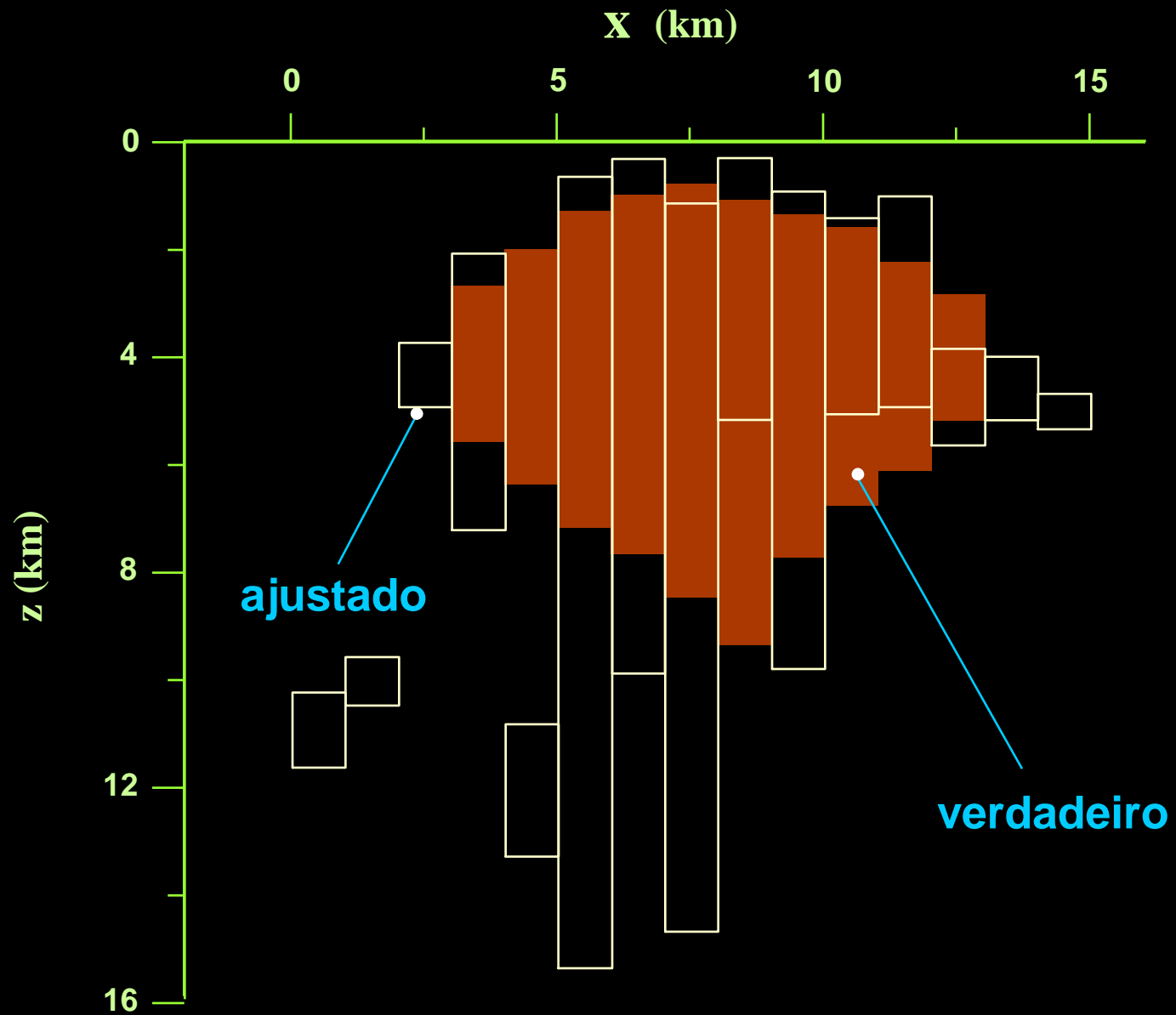


Batólito - suavidade

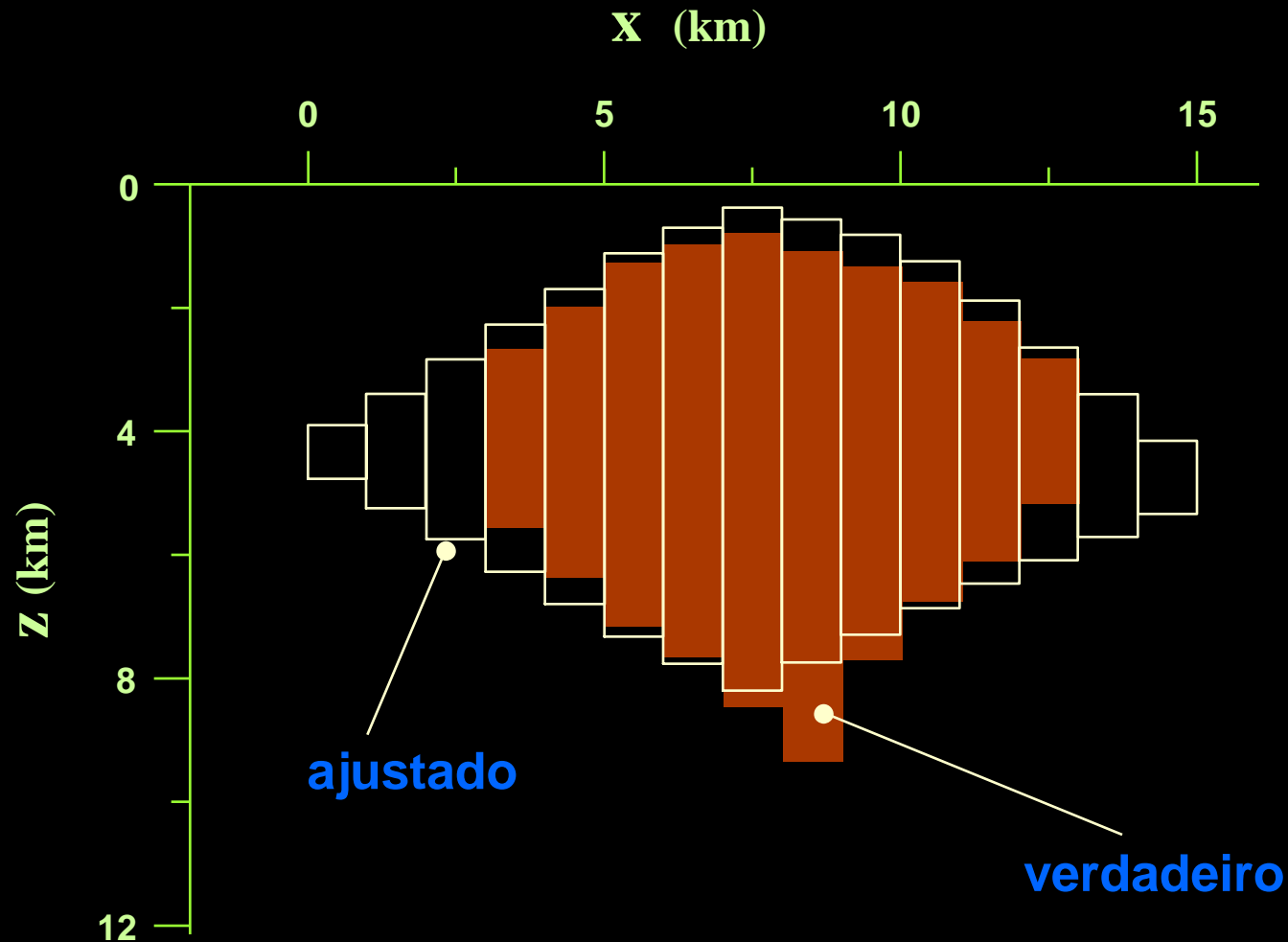
$$\lambda = 1,0$$



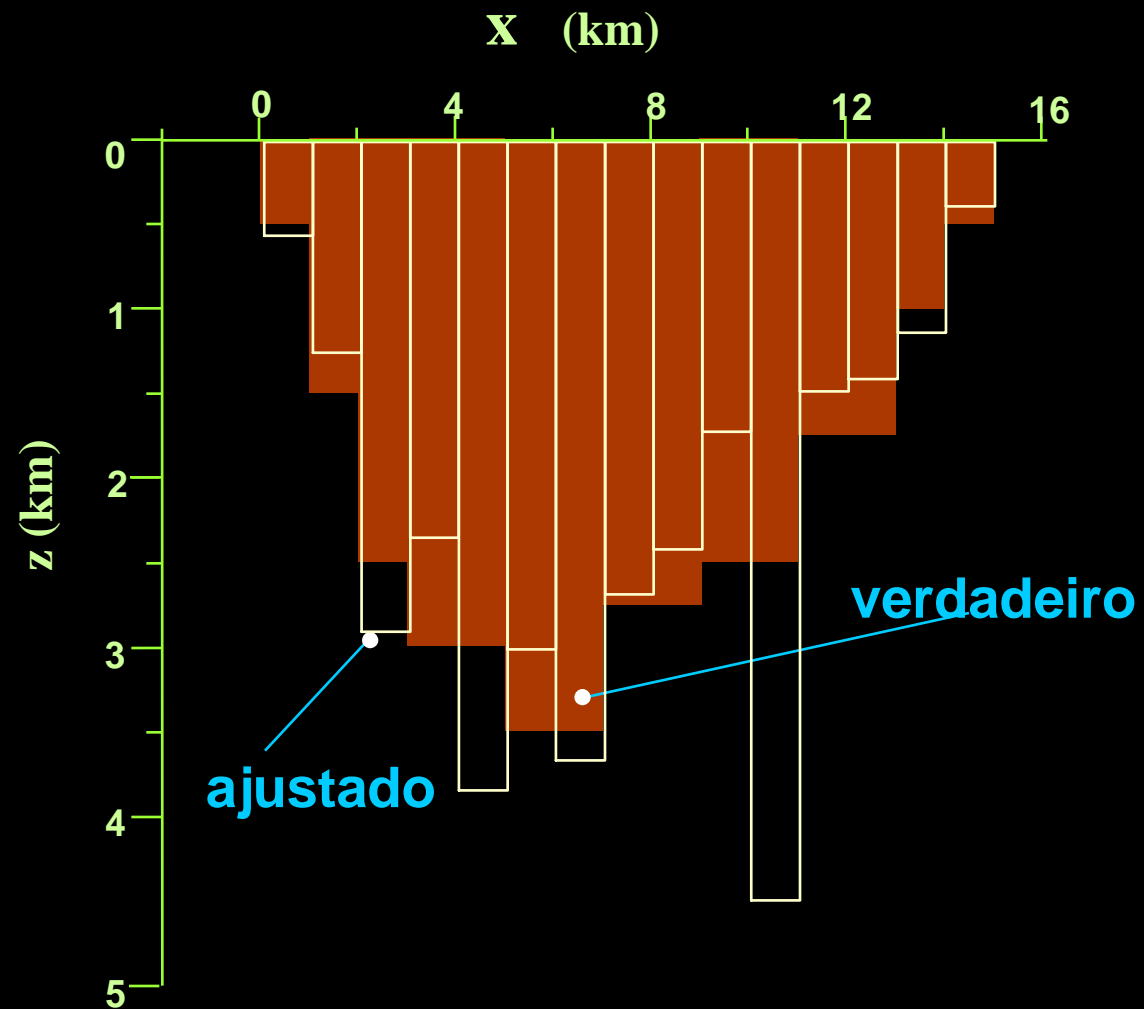
Batólito



Batólito - convexidade global

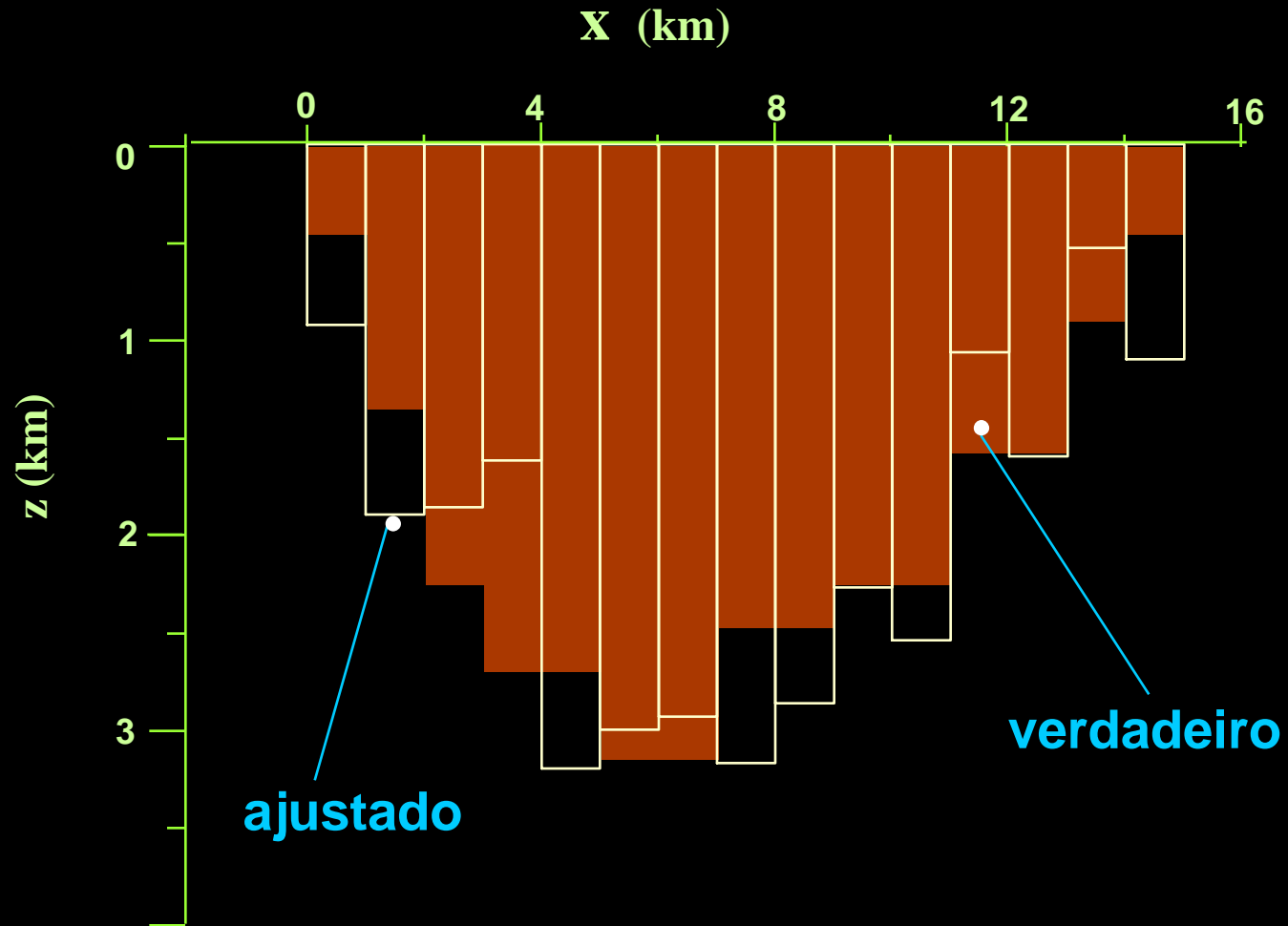


Bacia



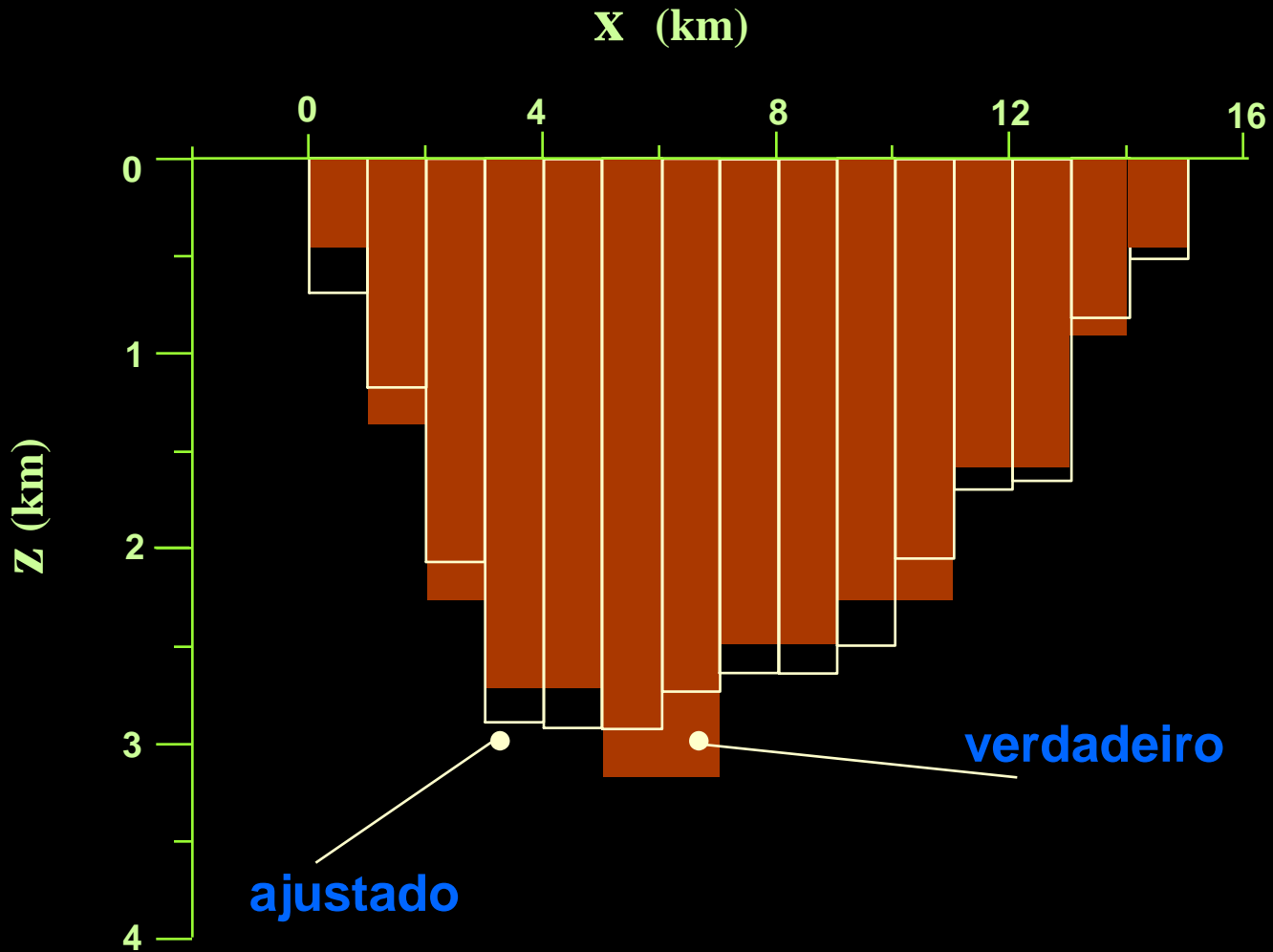
Bacia – convexidade em x

$d = 0.35 \text{ km}$



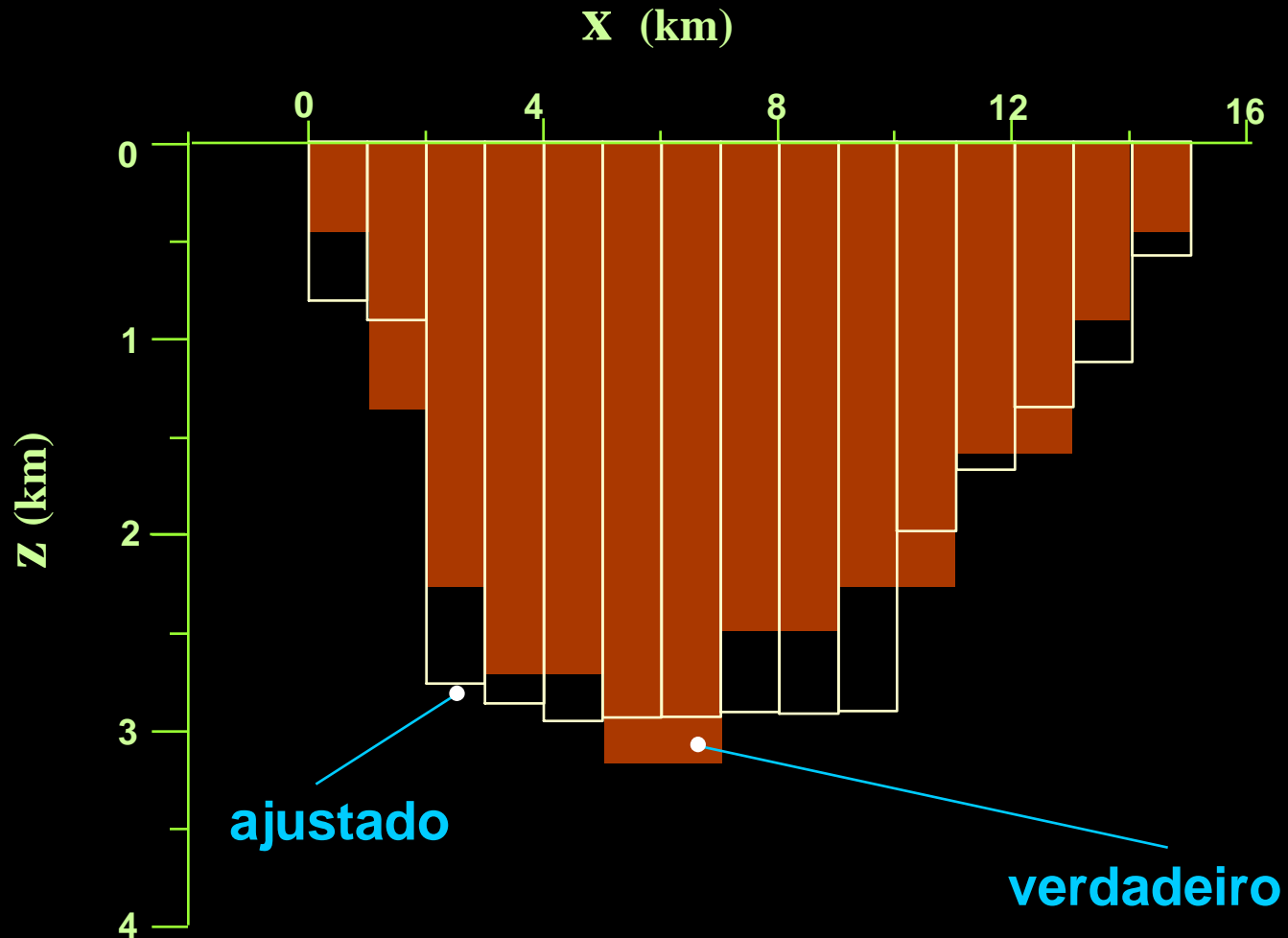
Bacia – convexidade em x

$d = 0.25 \text{ km}$



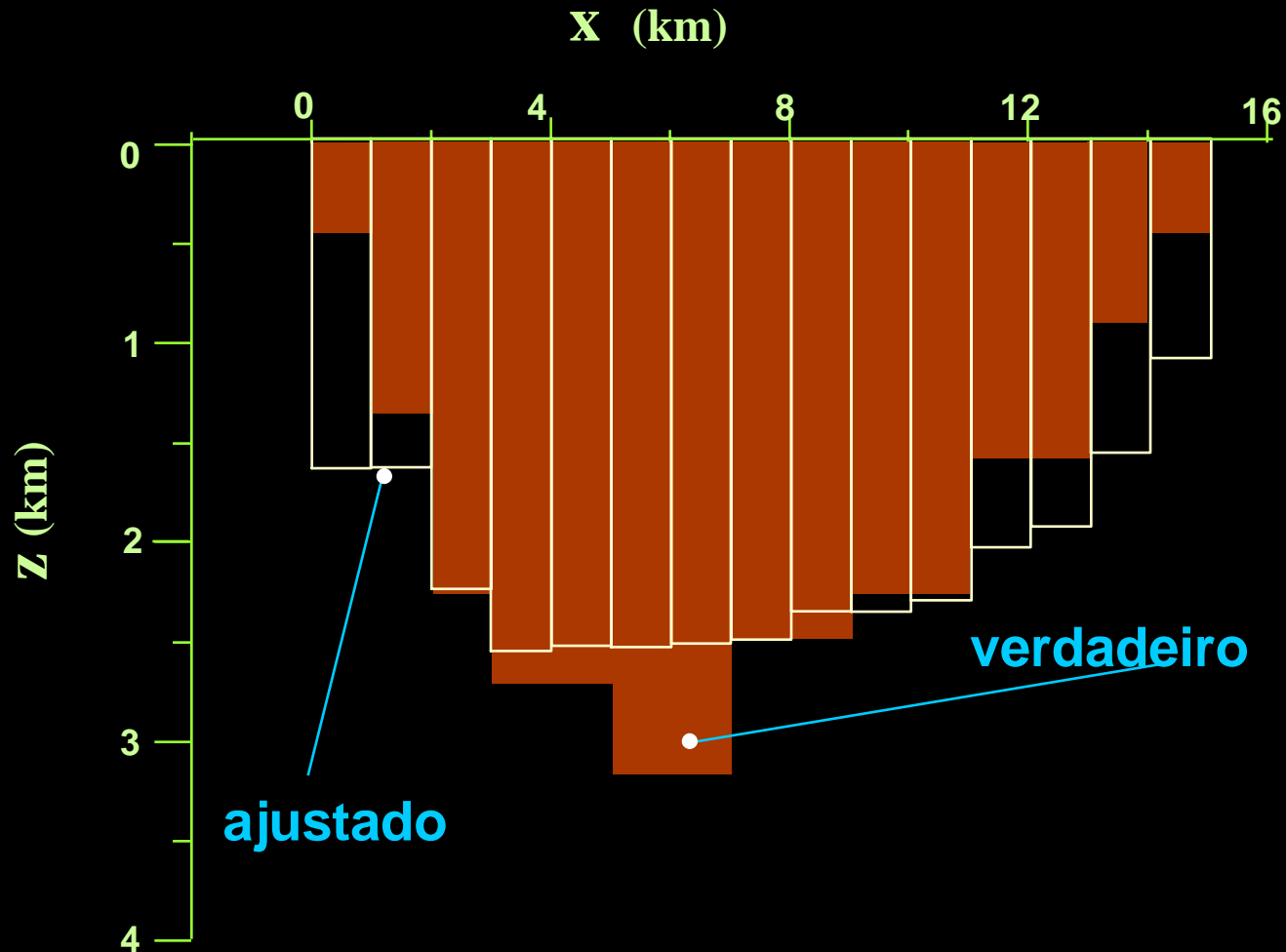
Bacia – convexidade em x

$d = 0.1 \text{ km}$



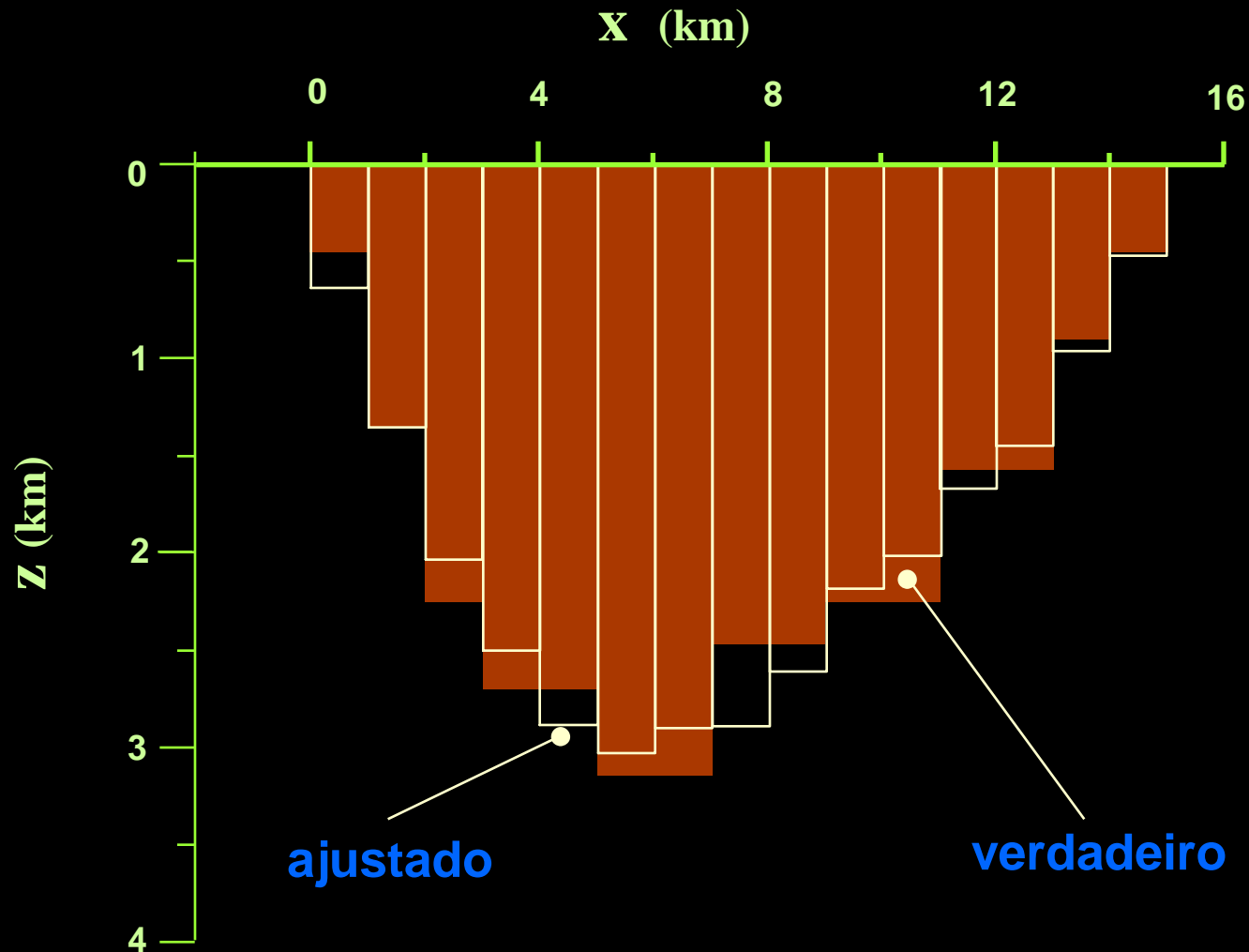
Bacia – convexidade em x

$d = 0.01 \text{ km}$



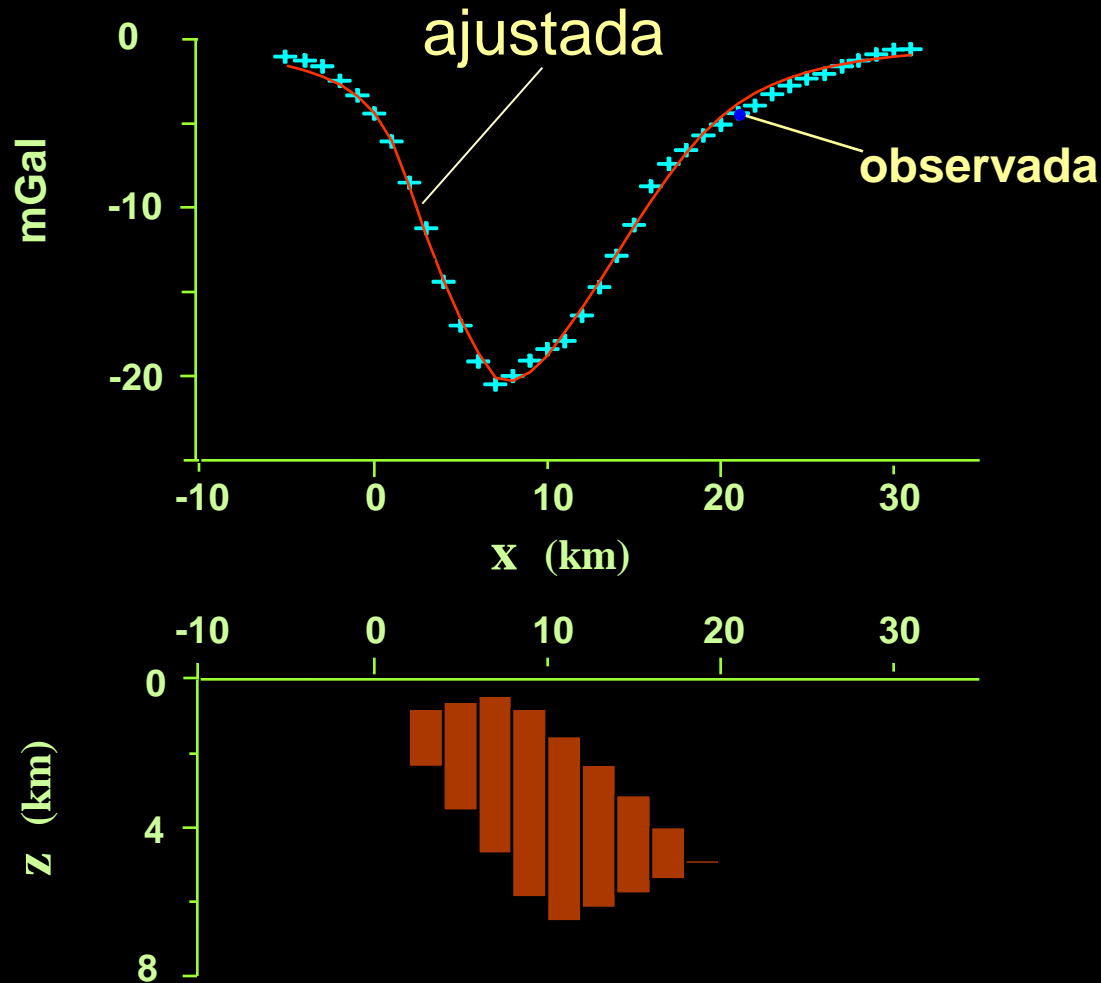
Suavidade - bacia

$$\lambda = 3.0$$

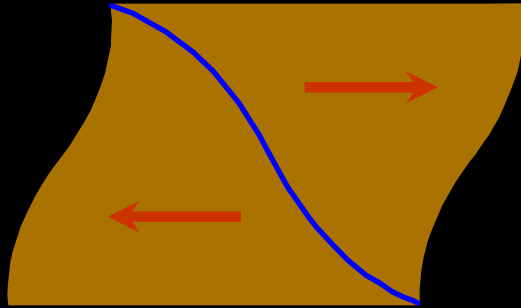


Granito de Castelsarrasin

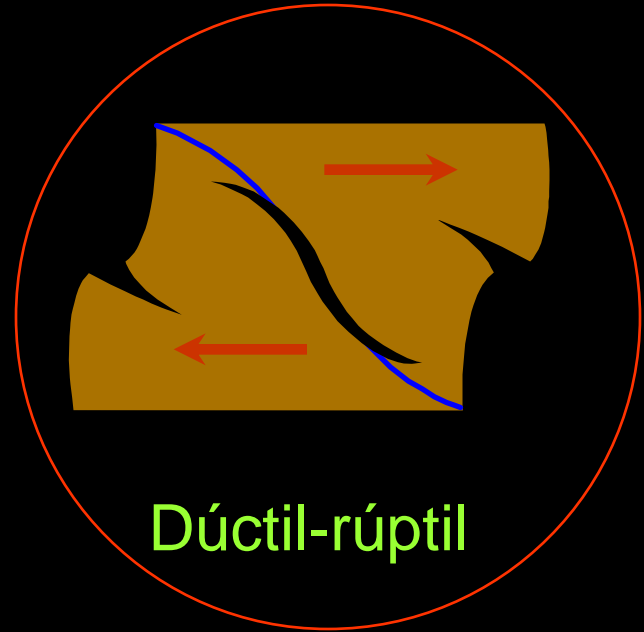
Bacia de Aquitaine, França



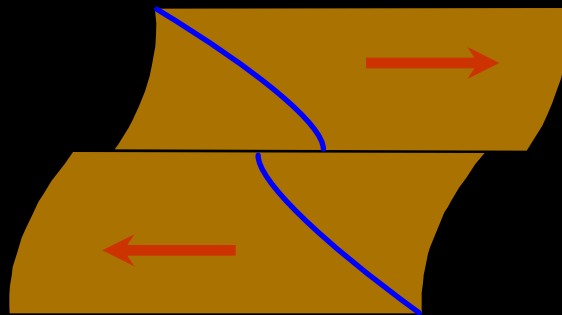
Tipos de deformação



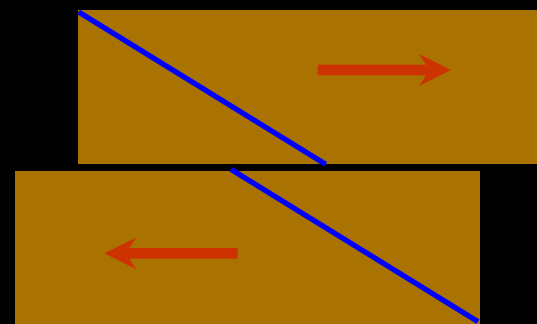
Dúctil



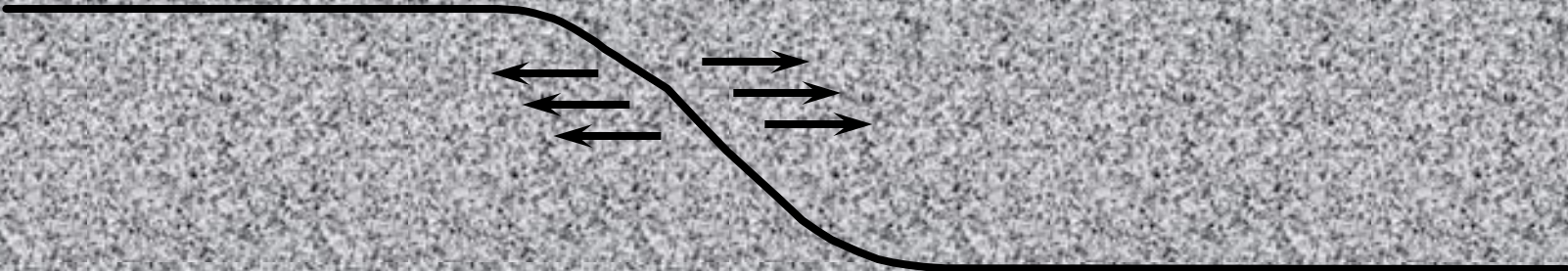
Dúctil-rúptil

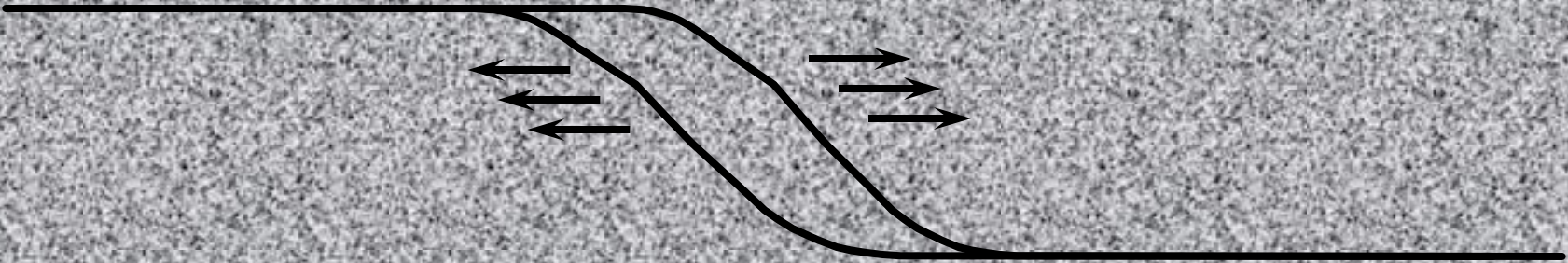


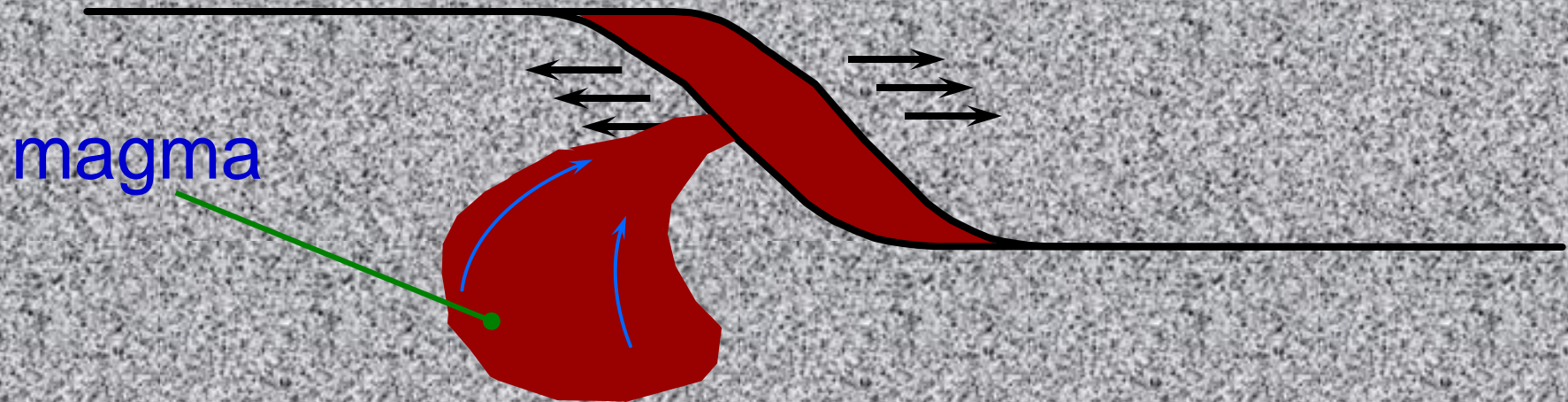
Rúptil-dúctil

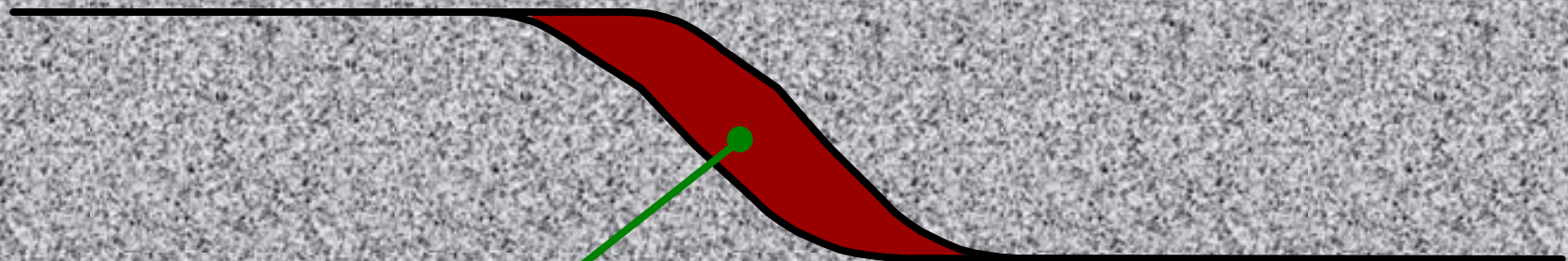


Rúptil





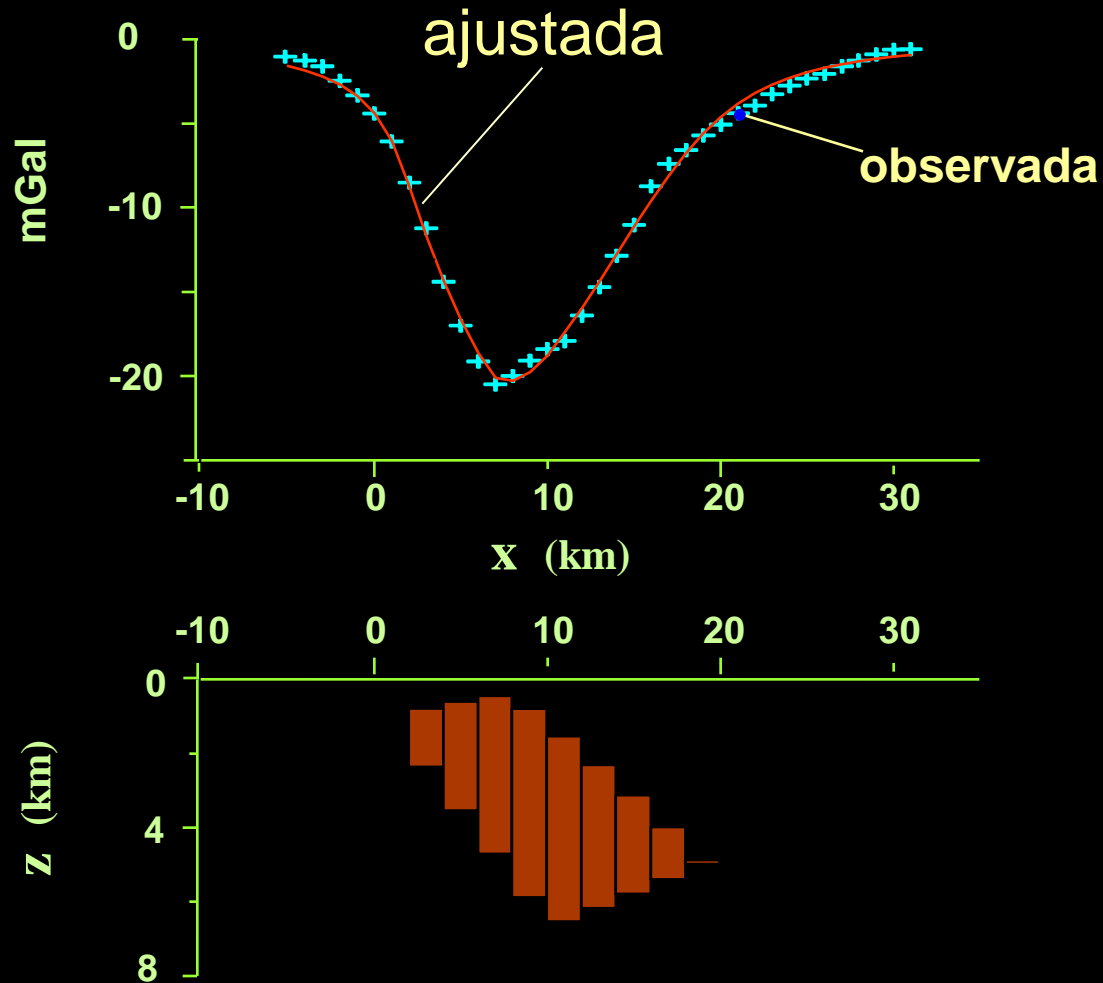


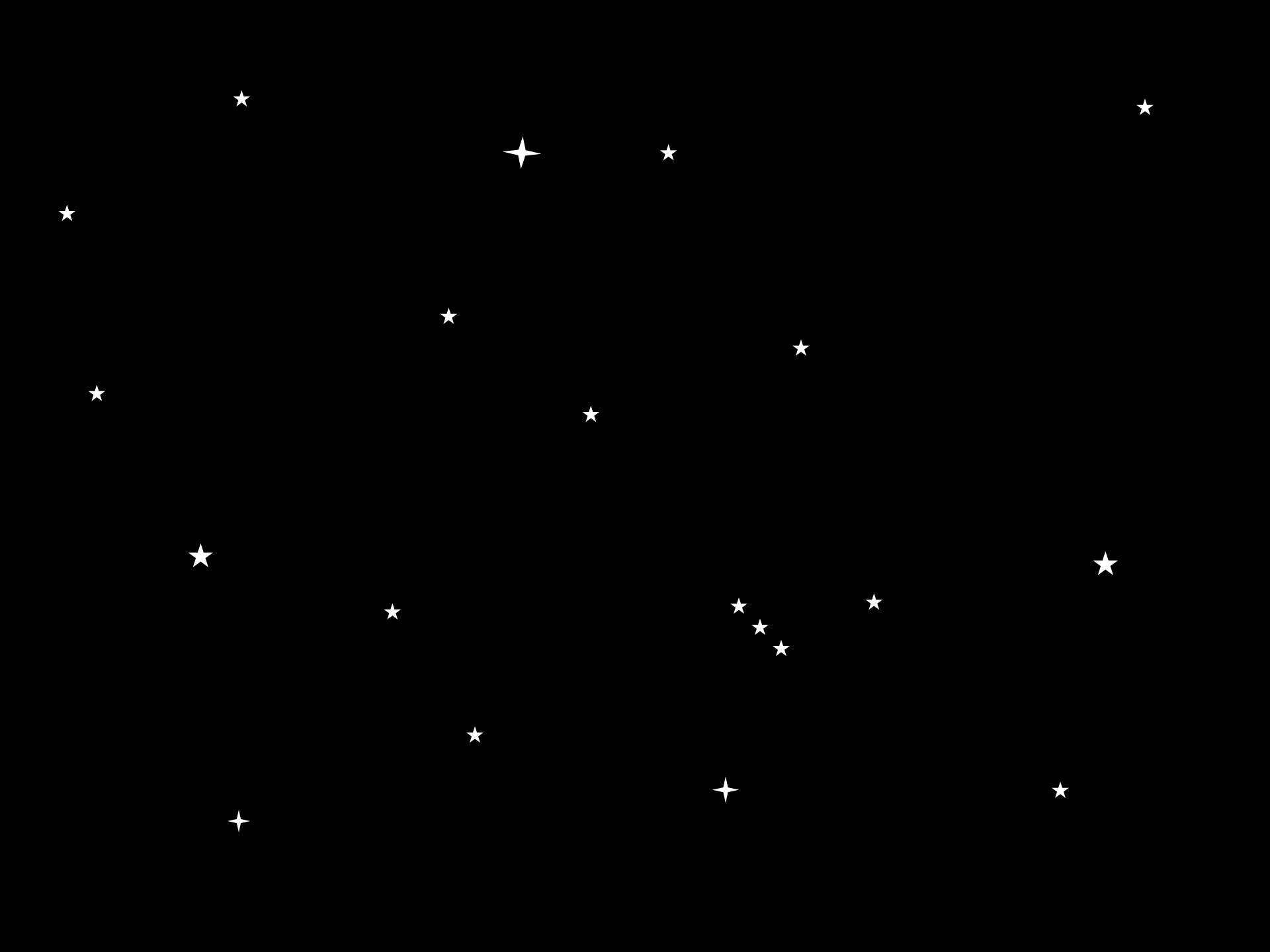


Corpo com forma sigmoidal

Granito de Castelsarrasin

Bacia de Aquitaine, França





Funcionais estabilizadores: (Tikhonov)


- Igualdade absoluta: “Ridge” suavdxy suavdxz suavhxy
- Igualdade relativa: Suavidade suavhxz suavdxy suavdxz suavhxy suavphxy
- Mínimo momento de inércia: compdxz
- Compacidade: compdcxz

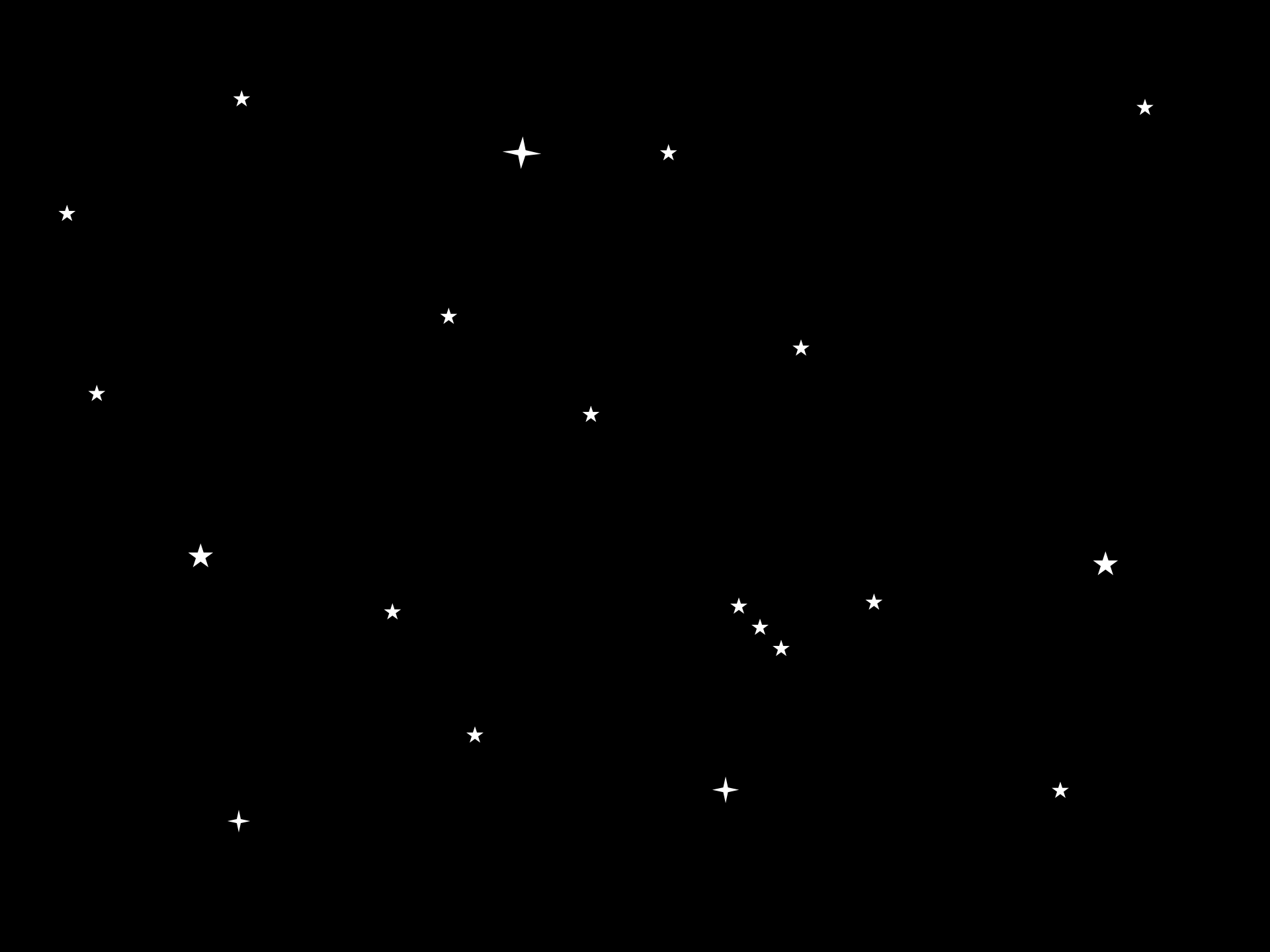
Funcionais estabilizadores:

- Convexidade global

convext

Funcionais e vínculos não estabilizadores:

- Desigualdade suavdxy suavphxy
 - Igualdade relativa ponderada suavphxy
 - Convexidade direcional
- 

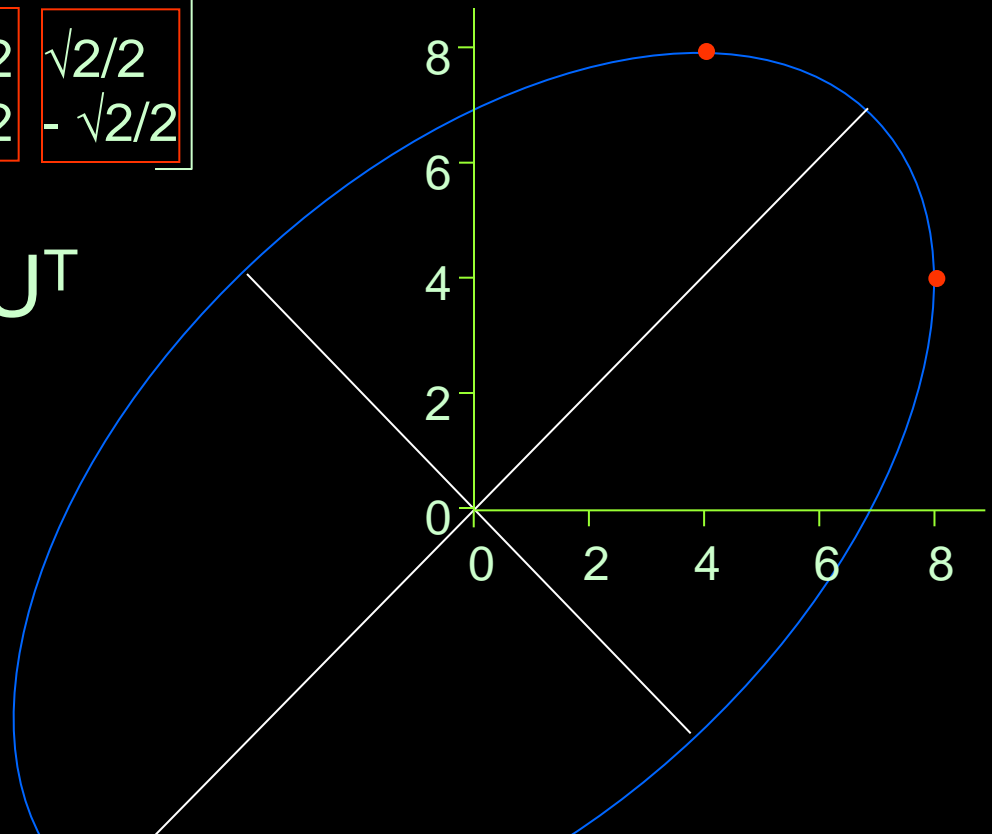


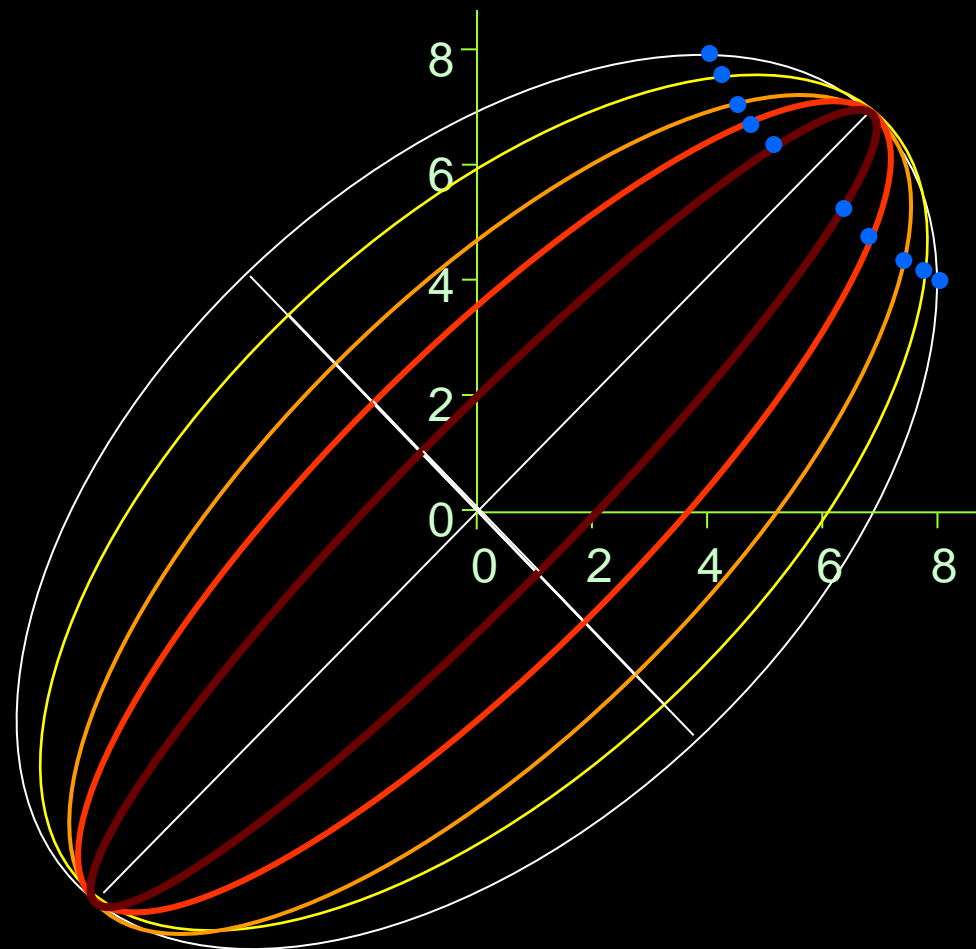
A inversa generalizada

A decomposição em valores singulares

$$A = \begin{bmatrix} 8 & 4 \\ 4 & 8 \end{bmatrix}$$

$$A = \underbrace{\begin{bmatrix} \sqrt{2}/2 & \sqrt{2}/2 \\ \sqrt{2}/2 & -\sqrt{2}/2 \end{bmatrix}}_U \underbrace{\begin{bmatrix} 12 & 0 \\ 0 & 4 \end{bmatrix}}_S \underbrace{\begin{bmatrix} \sqrt{2}/2 & \sqrt{2}/2 \\ \sqrt{2}/2 & -\sqrt{2}/2 \end{bmatrix}}_{U^T}$$





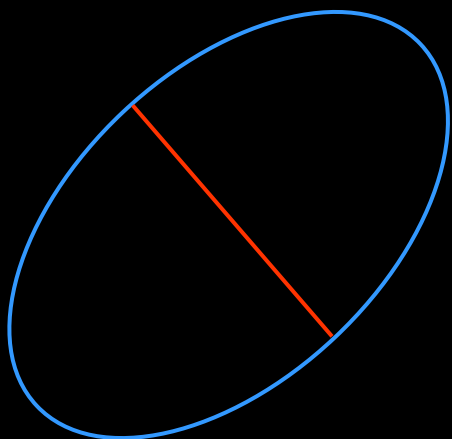
$$Q = \| \mathbf{U}^T \mathbf{y}^o - \mathbf{U}^T \mathbf{U} \mathbf{S} \mathbf{V}^T \mathbf{p} \|_2$$

$$Q = \| \mathbf{U}^T \mathbf{y}^o - \mathbf{S} \mathbf{V}^T \mathbf{p} \|_2$$

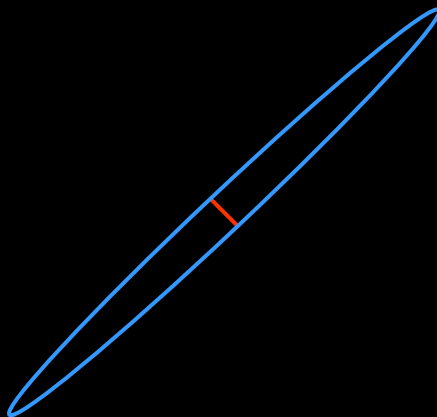
$$Q = \| \boldsymbol{\beta} - \mathbf{S} \boldsymbol{\alpha} \|_2$$

Três grupos de autovalores

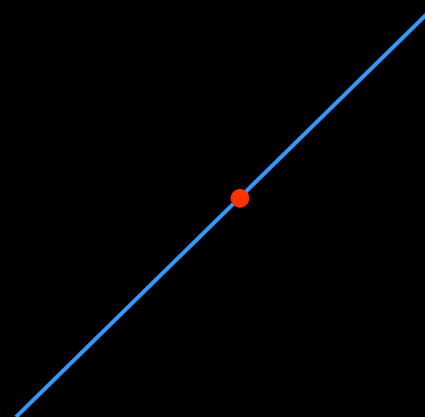
Os menores autovalores
são bem diferentes de zero



Os menores autovalores
são próximos a zero



Os menores autovalores
são iguais a zero



$$\mathbf{A} = \mathbf{U}\mathbf{S}\mathbf{V}^T$$

$$\mathbf{S} = \begin{bmatrix} \mathbf{S}_1 & 0 \\ 0 & \mathbf{S}_2 \\ 0 & 0 \end{bmatrix}$$

$$Q = \|\beta - \mathbf{S}\alpha\|_2^2 = \left\| \begin{bmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \end{bmatrix} - \begin{bmatrix} \mathbf{S}_1 & 0 \\ 0 & \mathbf{S}_2 \\ 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix} \right\|_2^2 = \left\| \begin{bmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \end{bmatrix} - \begin{bmatrix} \mathbf{S}_1 \alpha_1 \\ \mathbf{S}_2 \alpha_2 \\ 0 \end{bmatrix} \right\|_2^2$$

$$Q = \left\| \begin{bmatrix} \beta_1 - \mathbf{S}_1 \alpha_1 \\ \beta_2 - \mathbf{S}_2 \alpha_2 \\ \beta_3 \end{bmatrix} \right\|_2^2$$

$$Q = \left\| \beta_1 - \mathbf{S}_1 \alpha_1 \right\|_2^2 + \left\| \beta_2 - \mathbf{S}_2 \alpha_2 \right\|_2^2 + \left\| \beta_3 \right\|_2^2$$

$$\min_p Q = \underbrace{\|\beta_1 - S_1 \alpha_1\|}_{\text{Autovalores bem diferentes de zero}} + \underbrace{\|\beta_2 - S_2 \alpha_2\|}_{\text{Autovalores próximos a zero}} + \|\beta_3\|$$

Autovalores bem
diferentes de zero

Autovalores
próximos a zero

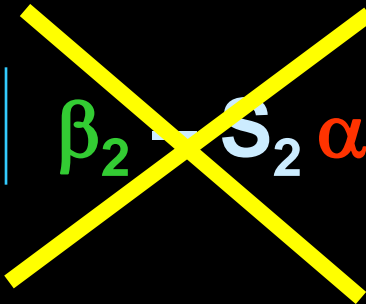
$$\beta_1 = U_1^T y^o$$

$$\alpha_1 = V_1^T p$$

$$\beta_2 = U_2^T y^o$$

$$\alpha_2 = V_2^T p$$

$$A = USV^T$$

$$\min_p \quad \left\| \beta_1 - S_1 \alpha_1 \right\| + \left\| \beta_2 - S_2 \alpha_2 \right\| + \left\| \beta_3 \right\|$$


$$\beta_1 - S_1 \alpha_1 = 0 \quad \longrightarrow \quad \alpha_1 = S_1^{-1} \beta_1$$

$$p = V_1 \alpha_1 + V_2 \alpha_2 \quad \alpha_2 = ?$$

INVERSA GENERALIZADA: $\alpha_2 = 0$

$$p = V_1 \alpha_1$$

$$p = V_1 S_1^{-1} \beta_1$$

$$p = V_1 S_1^{-1} U_1^T y^o$$

$$\mathbf{p} = \mathbf{V}_1 \mathbf{S}_1^{-1} \mathbf{U}_1^T \mathbf{y}^o$$

$$\mathbf{p} = \mathbf{V}_1 \alpha_1 + \mathbf{V}_2 \alpha_2$$

$$\mathbf{p} = \mathbf{V} \alpha$$

$$\|\mathbf{p}\| = \mathbf{p}^T \mathbf{p} = \alpha^T \mathbf{V}^T \mathbf{V} \alpha = \alpha^T \alpha$$

$$\alpha^T \alpha = \begin{bmatrix} \alpha_1^T & \alpha_2^T \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix} = \alpha_1^T \alpha_1 + \alpha_2^T \alpha_2$$

$$\alpha_2 = 0$$



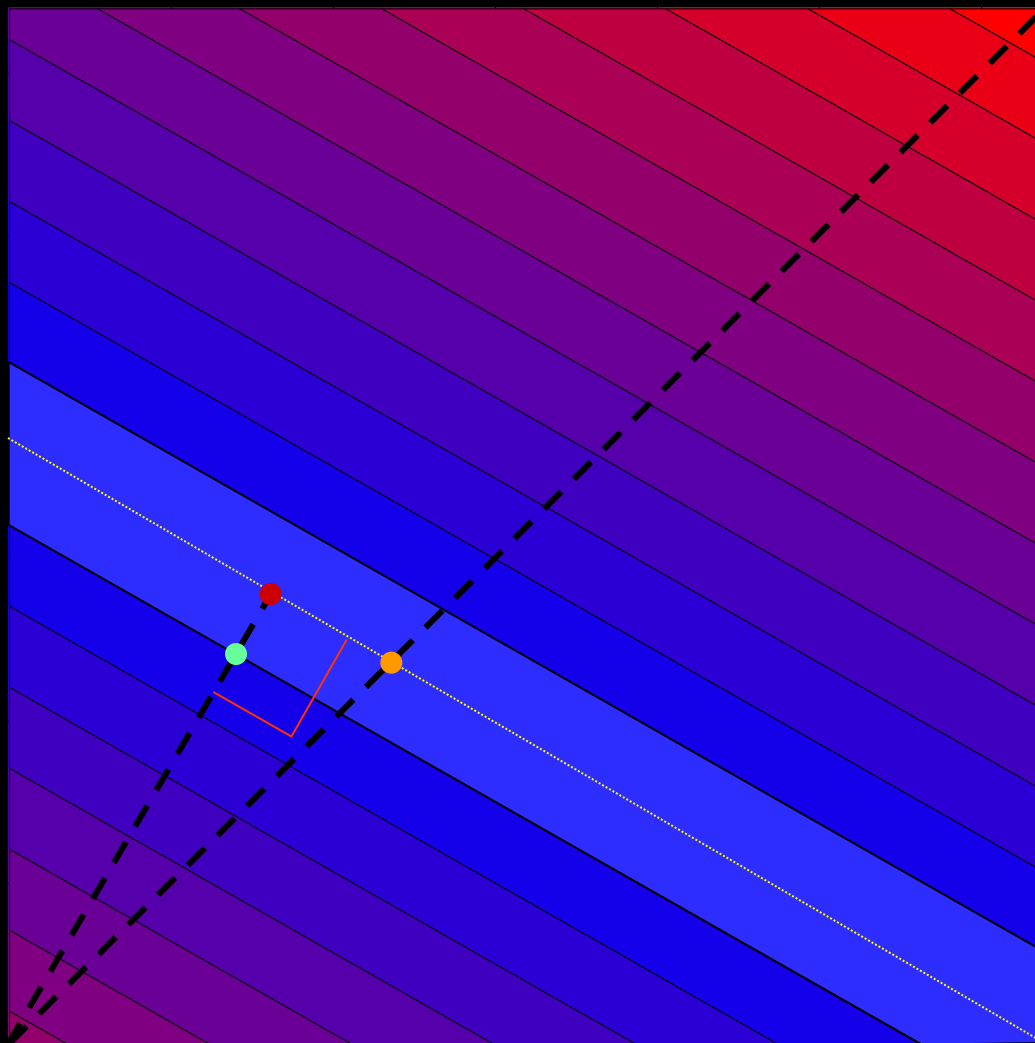
Solução de menor norma euclideana

Em termos da informação a priori introduzida para estabilizar o problema inverso:

Inversa generalizada \equiv ridge \equiv suavidade

Inversa Suavizada Generalizada

p_2



p_1

