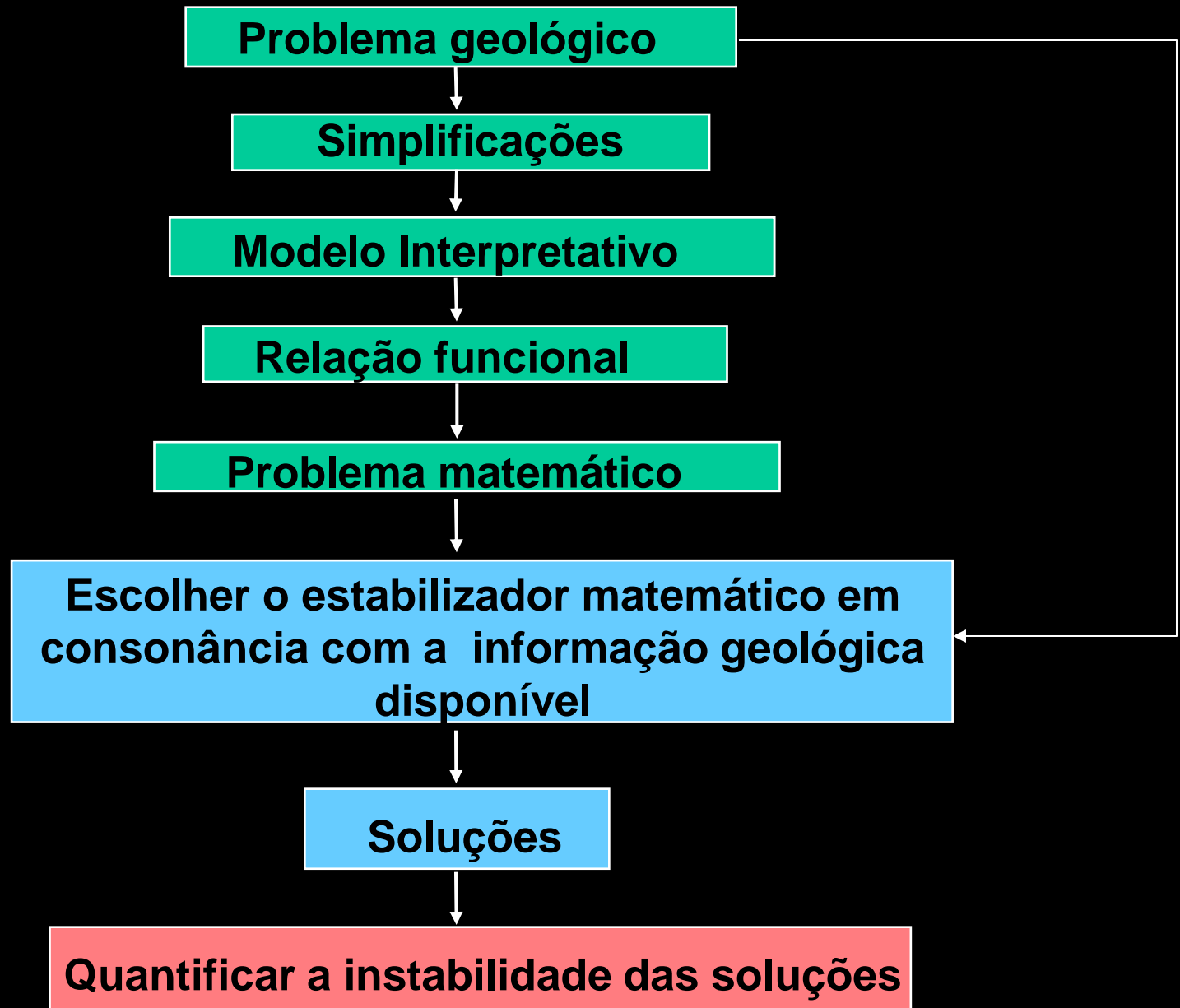
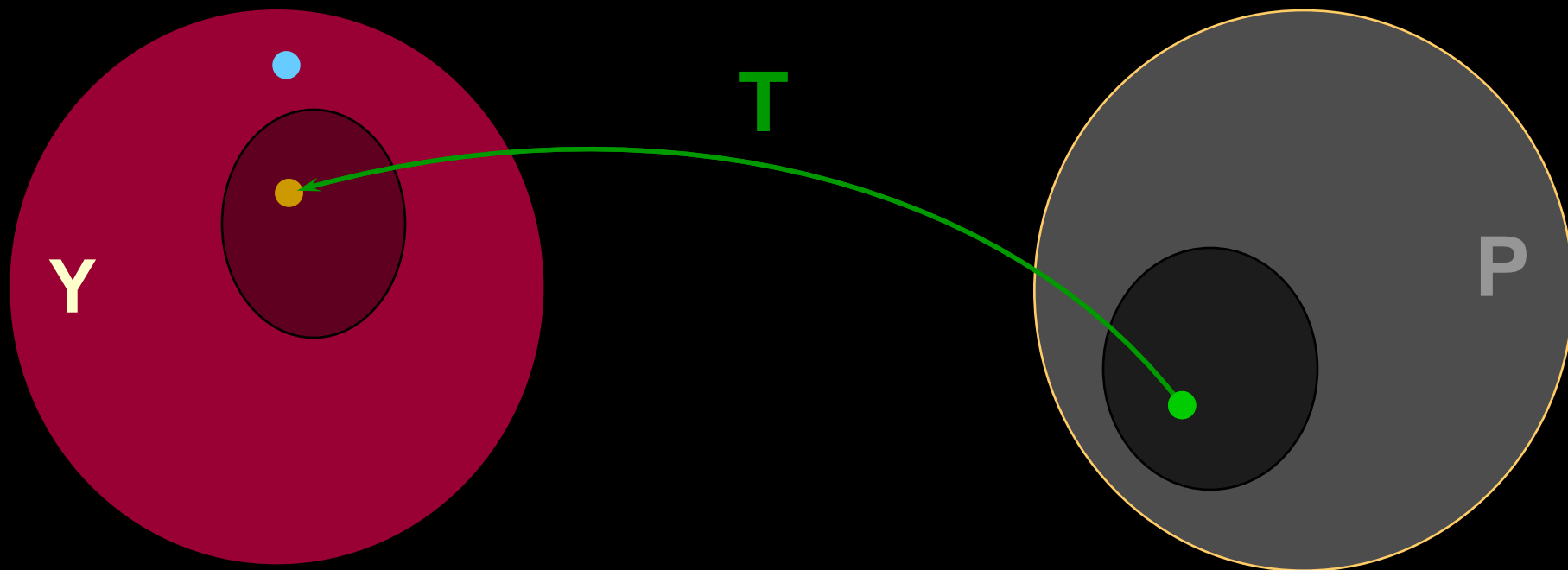
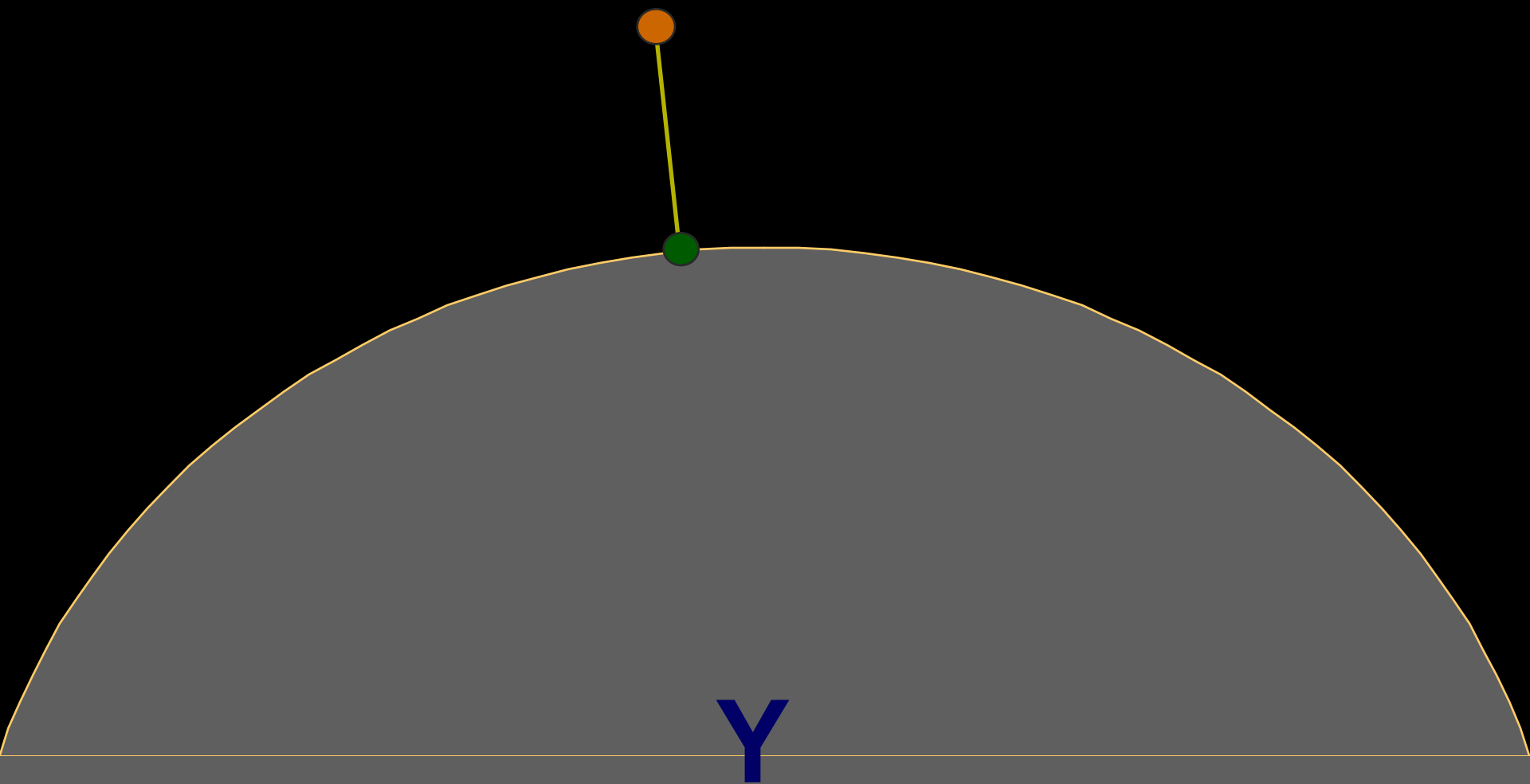


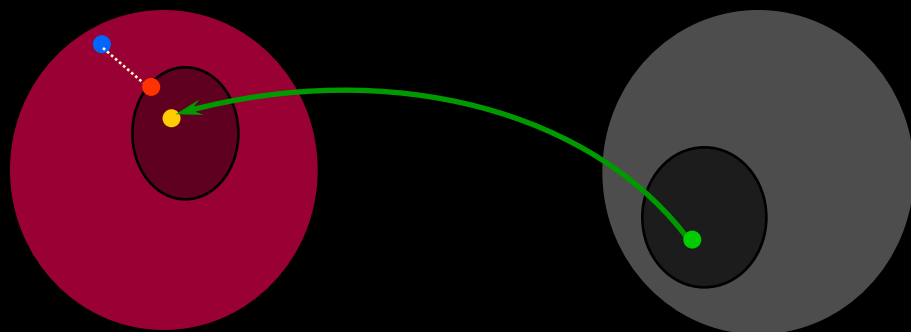
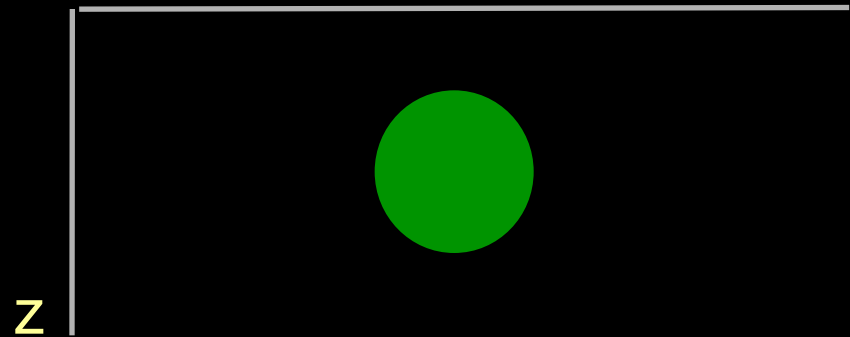
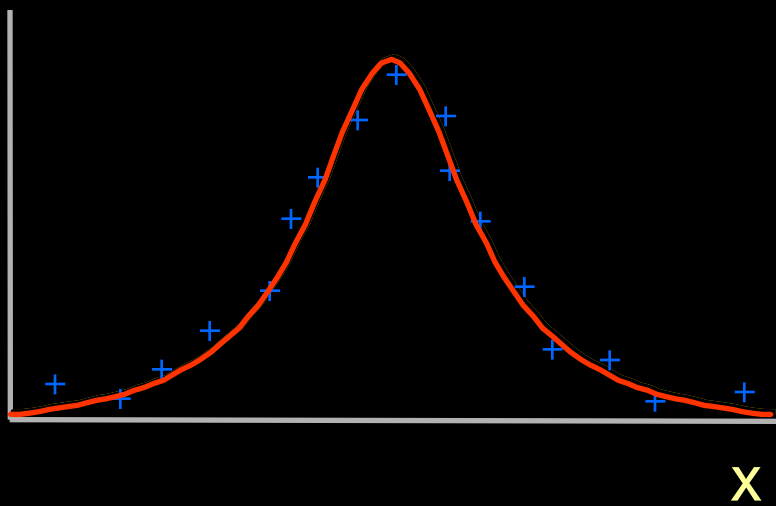
- EXISTÊNCIA
- UNICIDADE
- ESTABILIDADE



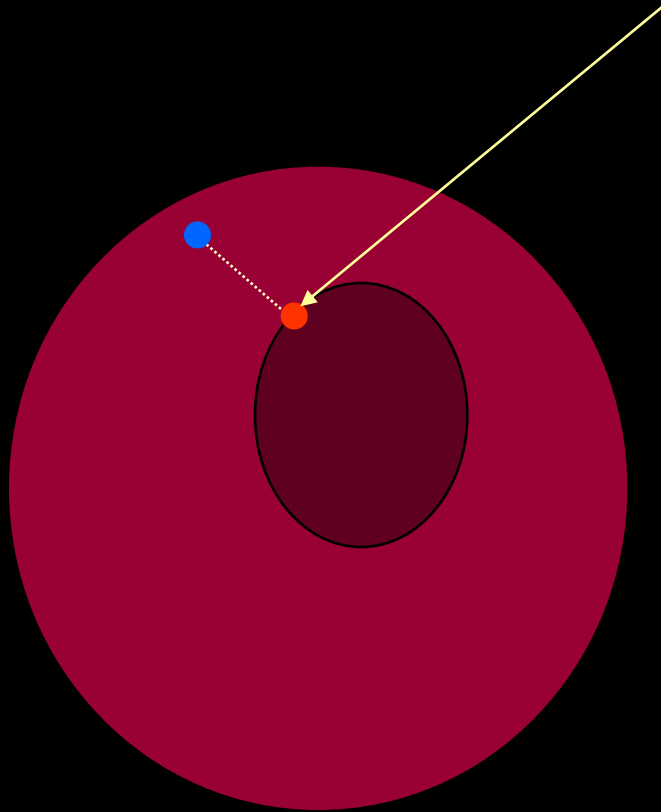


$$Y = T\{P\}$$

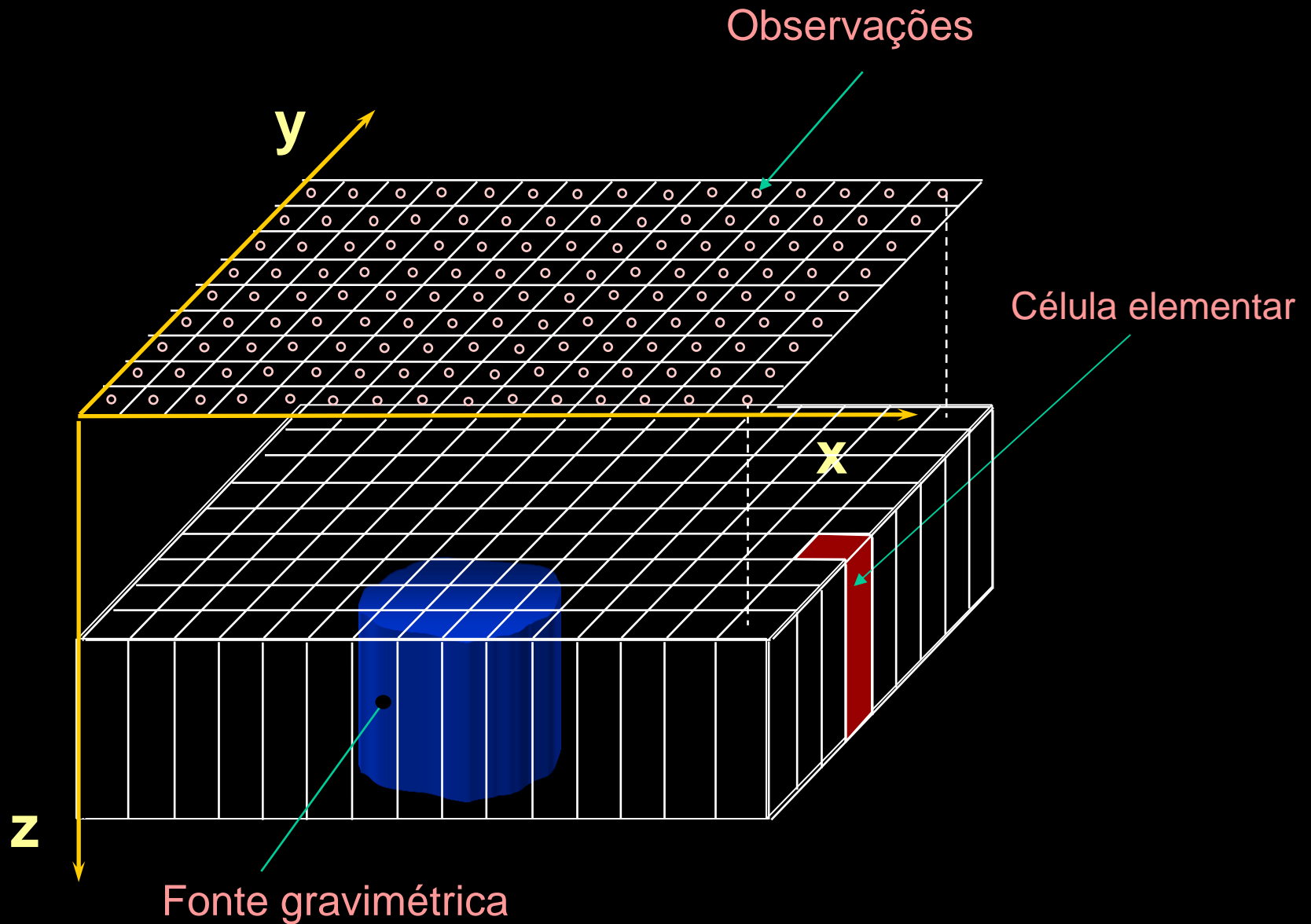




# Solução de mínimos quadrados



# Problema geofísico linear



# Problema matemático:

Resolver um sistema de equações lineares

$$A p = y^o$$



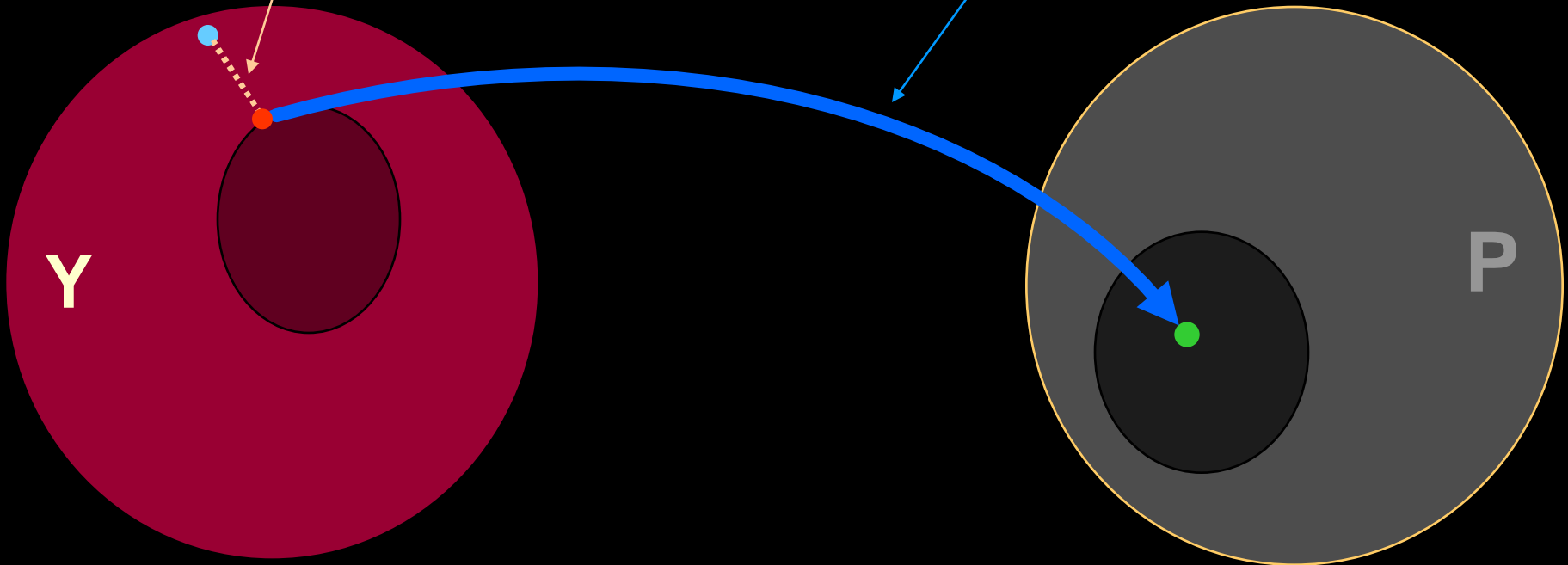
Em geral a solução exata não existe

A solução de mínimos quadrados é dada por:

$$\hat{\mathbf{p}} = \left( \mathbf{A}^T \mathbf{A} \right)^{-1} \mathbf{A}^T \mathbf{y}^o$$

Mínimos cuadrados:  $A^T A p = A^T y^o$

Operador inverso:  $p = (A^T A)^{-1} A^T y^o$



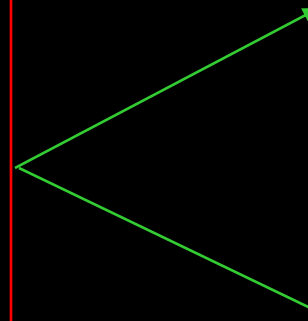
- EXISTÊNCIA

- UNICIDADE

- ESTABILIDADE

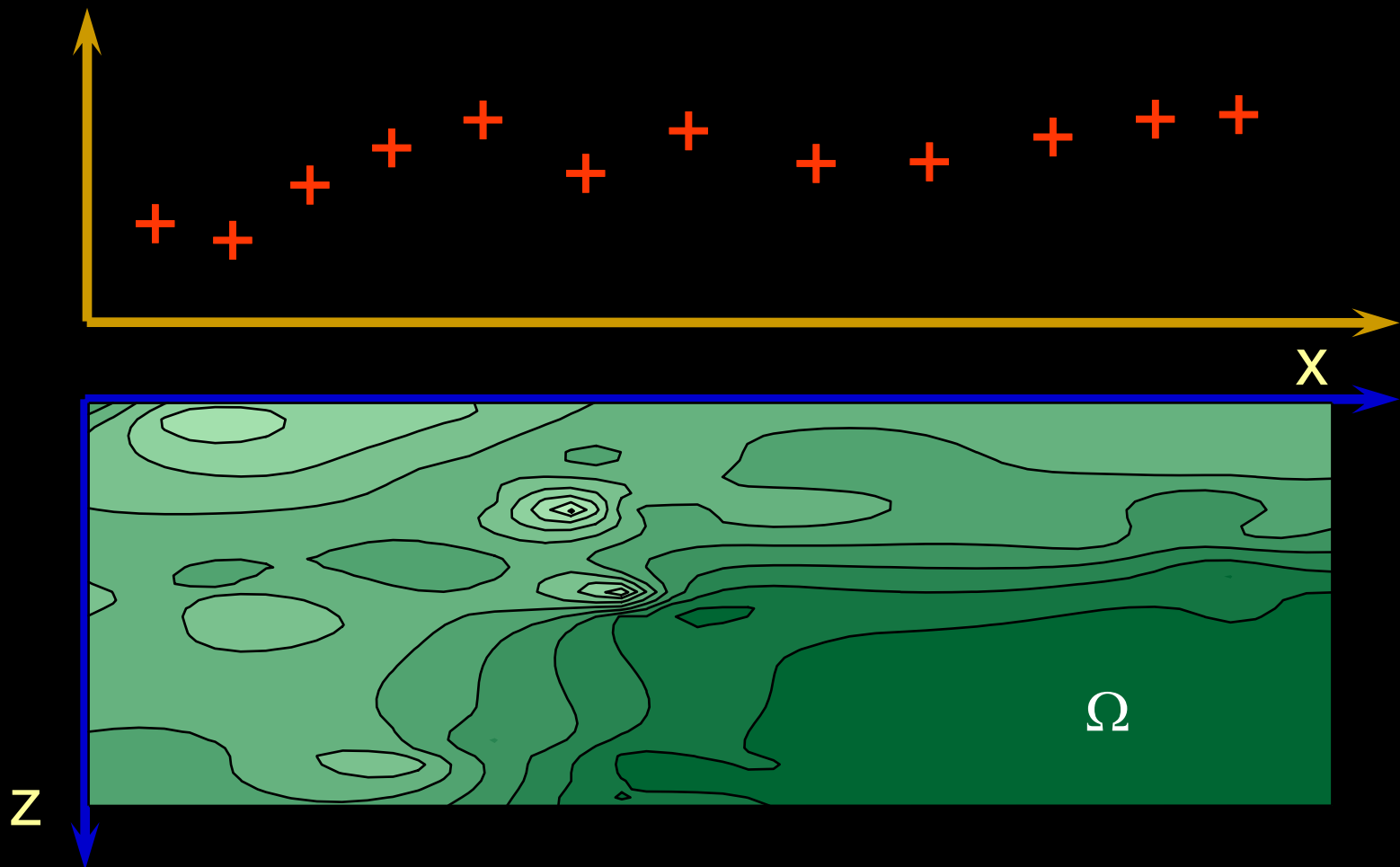
Backus-Gilbert

Tikhonov



# Teoria de Backus-Gilbert

$$y^o(x_i, z_i) = \int_{\Omega} G(x_i, z_i; x', z') p(x', z') dx' dz'$$



## Diagnóstico:

O problema inverso geofísico é subdeterminado e portanto não apresenta solução única

## Remédio:

Extrair dos dados apenas a informação neles contida

## Características comuns a todas as soluções:

- A solução menos suavizada possível
- O excesso de massa

$$C = \sum_{i=1}^N k_i \int_{\Omega} G(x_i - x') p(x') \, dx' dz'$$

$$C = \int_{\Omega} \sum_{i=1}^N k_i G(x_i - x') p(x') \, dx' dz'$$

$$C = \int_{\Omega} \left[ \sum_{i=1}^N k_i G(x_i - x') \right] p(x') \, dx' \, dz'$$

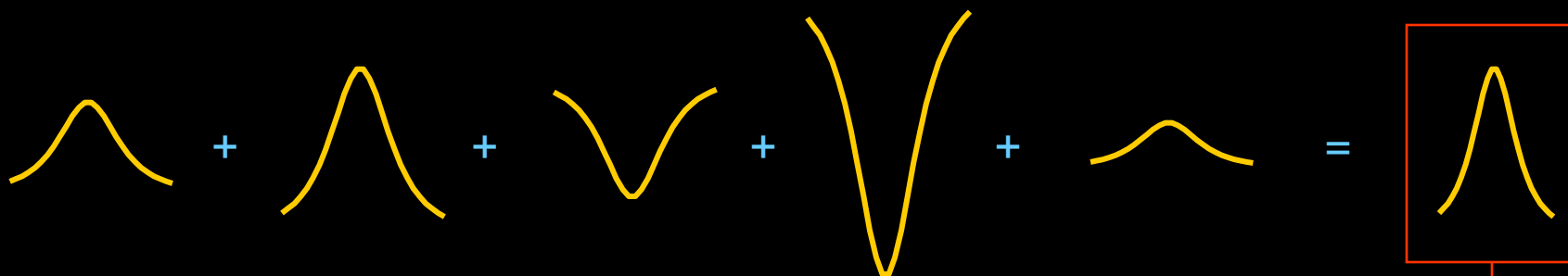
$$\equiv f(x_i - x')$$

$$C = \int_{\Omega} f(x_i - x') p(x') \, dx' \, dz'$$

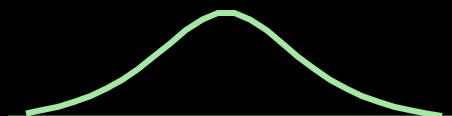
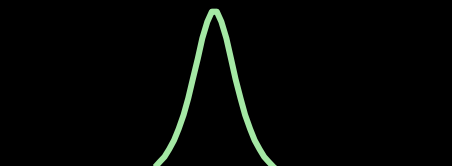
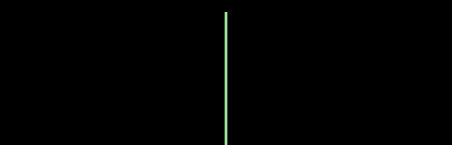
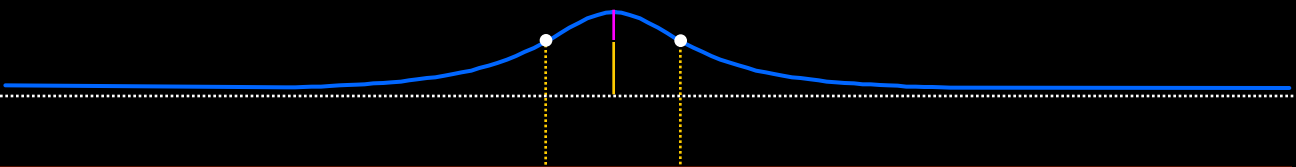
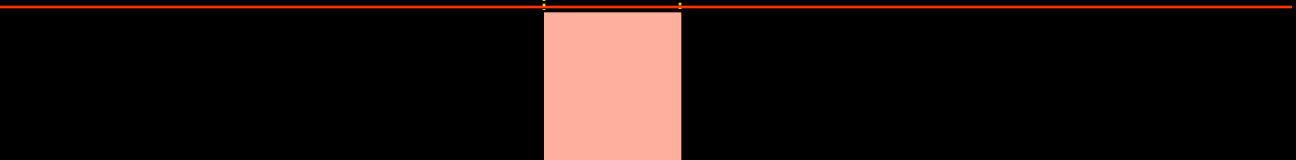
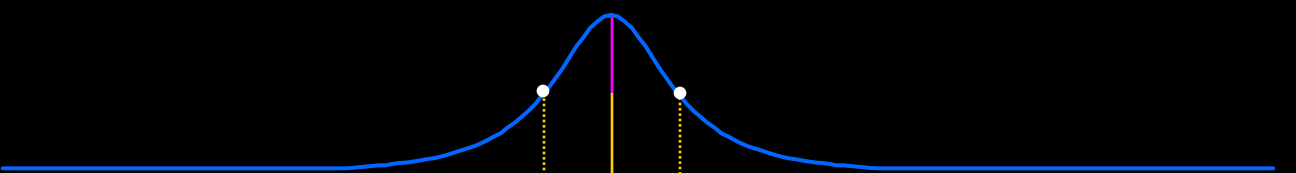
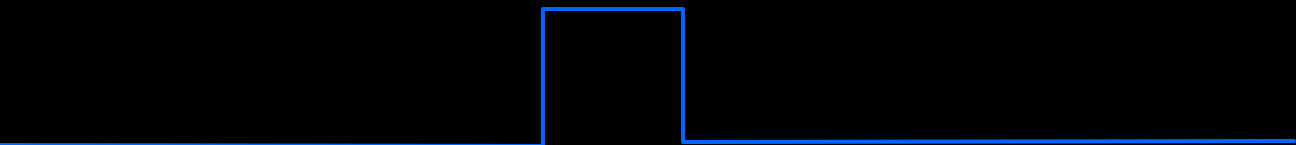
Se  $f(x_i - x') \approx \delta(x_i - x')$ :  $C = p(x_i)$



$$\sum_{i=1}^N k_i G(x_i - x') \approx \delta(x_i - x')$$



$$C = \int_{\Omega} \sum_{i=1}^N k_i G(x_i - x') p(x') dx' dz'$$

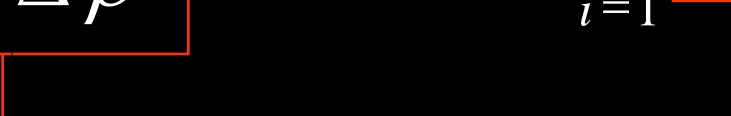


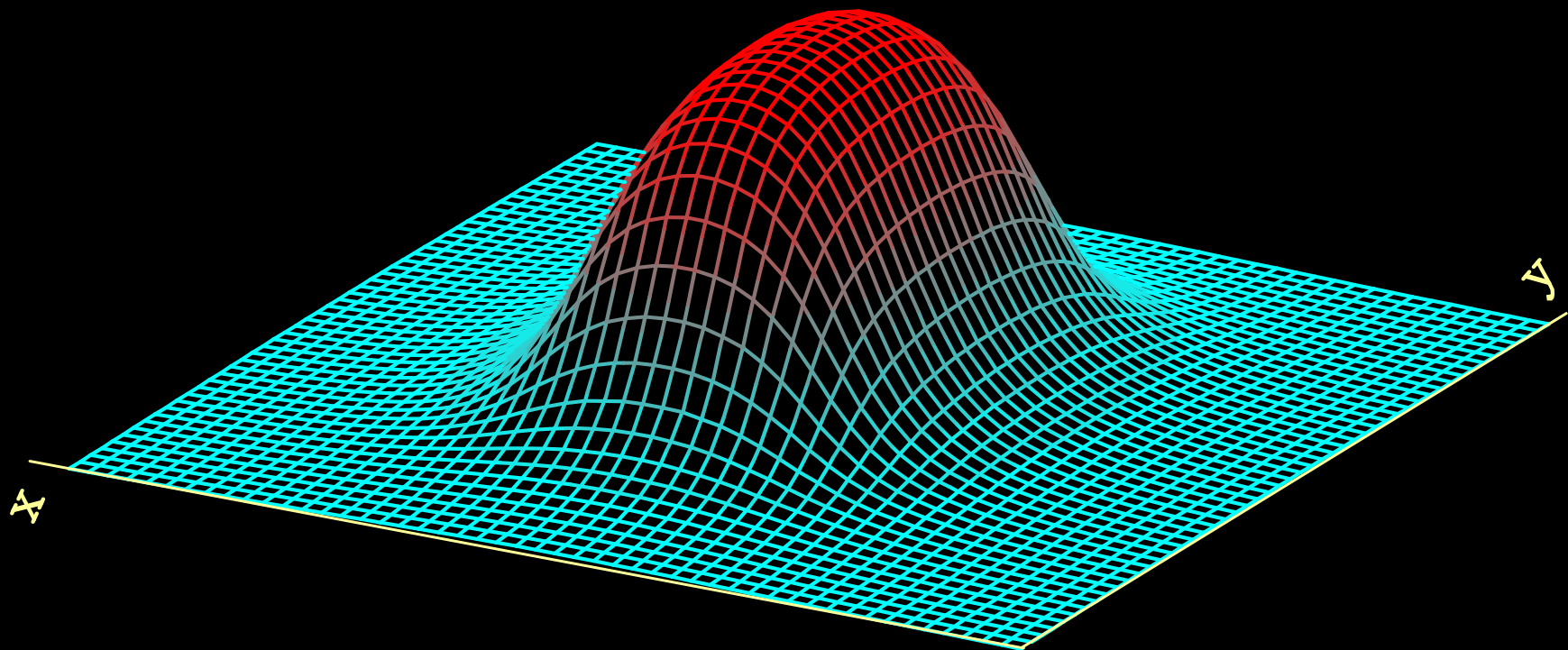
# Características comuns a todas as soluções:

- A solução menos suavizada possível
- O excesso de massa

$$M = \frac{1}{4\pi\gamma\Delta\rho} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} y^o(x', y') dx' dy'$$

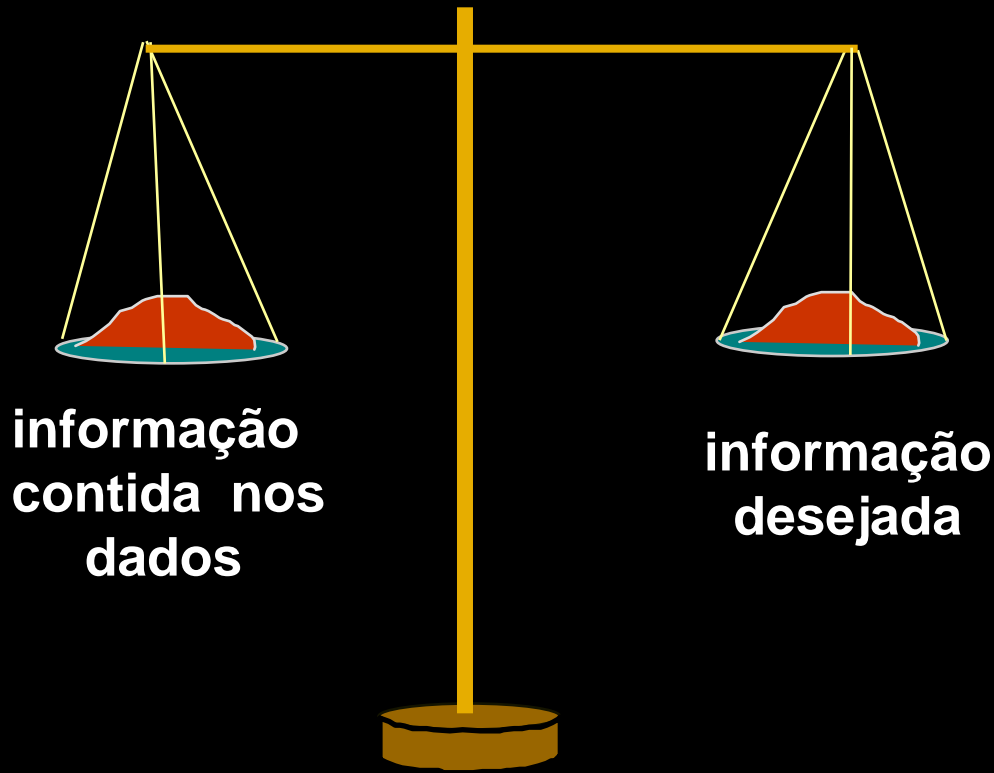
$$M = \sum_{i=1}^N \frac{1}{4\pi\gamma\Delta\rho} y_i^o$$

$$M = \sum_{i=1}^N \boxed{\frac{1}{4\pi\gamma\Delta\rho}} y_i^o = \sum_{i=1}^N \boxed{k_i} y_i^o$$




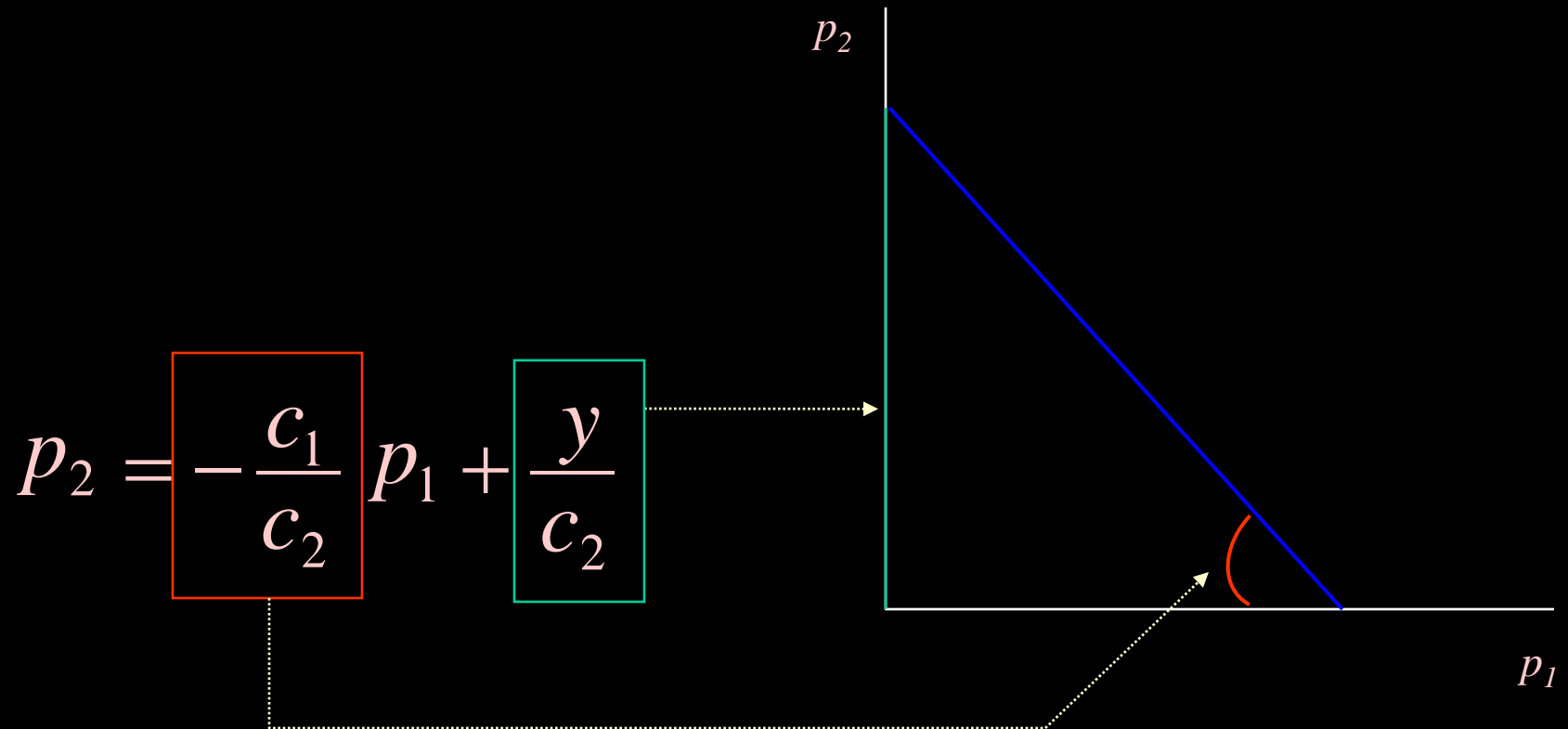
# Backus-Gilbert:

Reduzir a demanda de informação

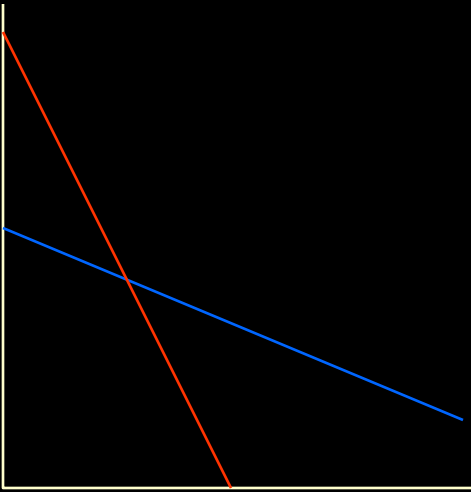


# Regularização de Tikhonov

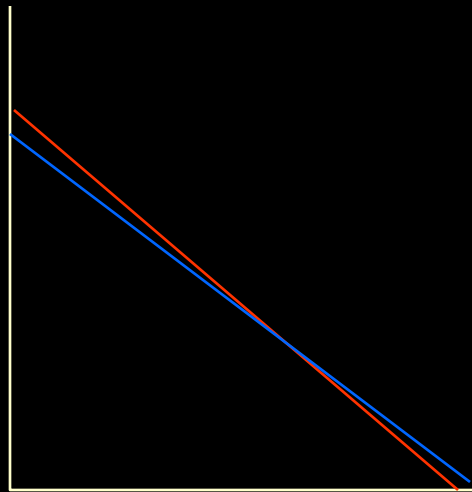
Paradigma de um problema geofísico linear:



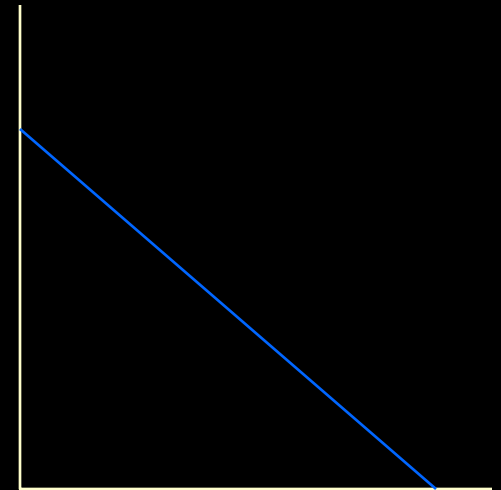
# Tipos de problema inverso



Solução estável



Solução instável

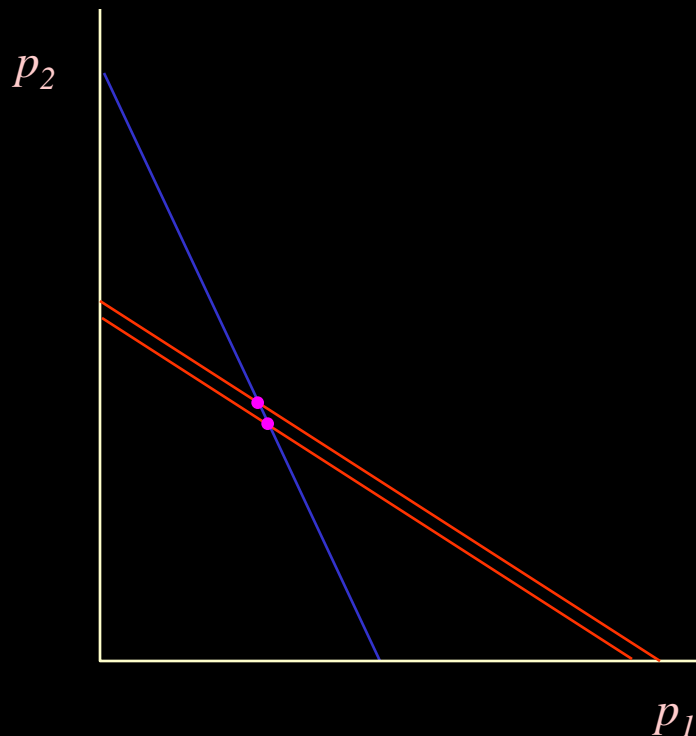


Solução não única

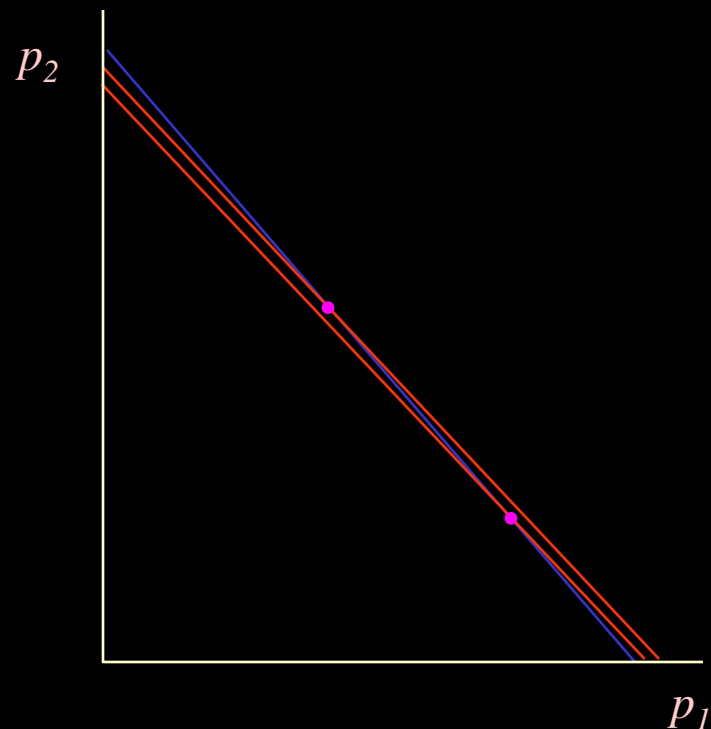


# Instabilidade

$$p_2 = -\frac{c_1}{c_2} p_1 + \frac{y}{c_2}$$

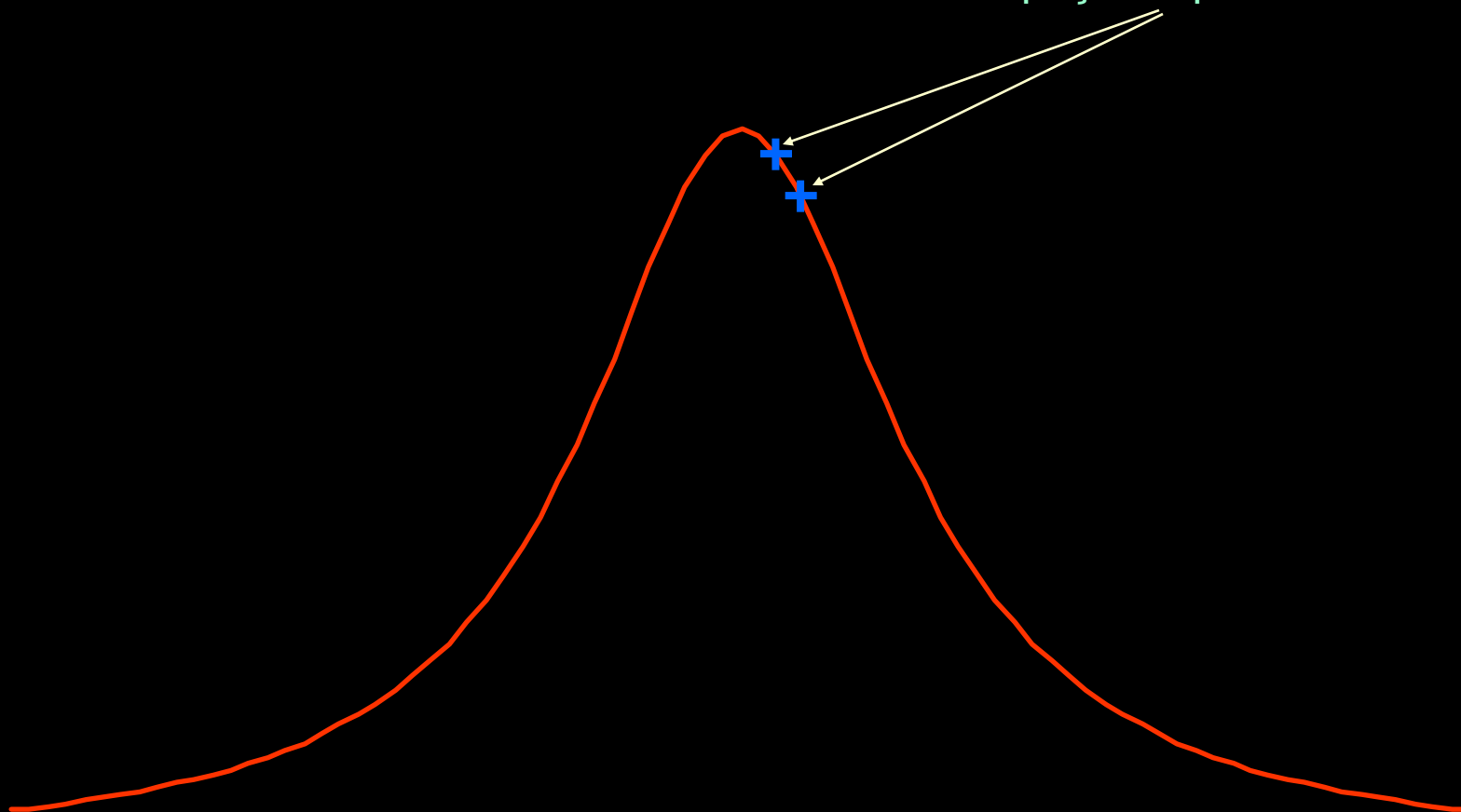


Solução estável



Solução instável

Observações redundantes:  
retas sub paralelas no  
espaço de parâmetros



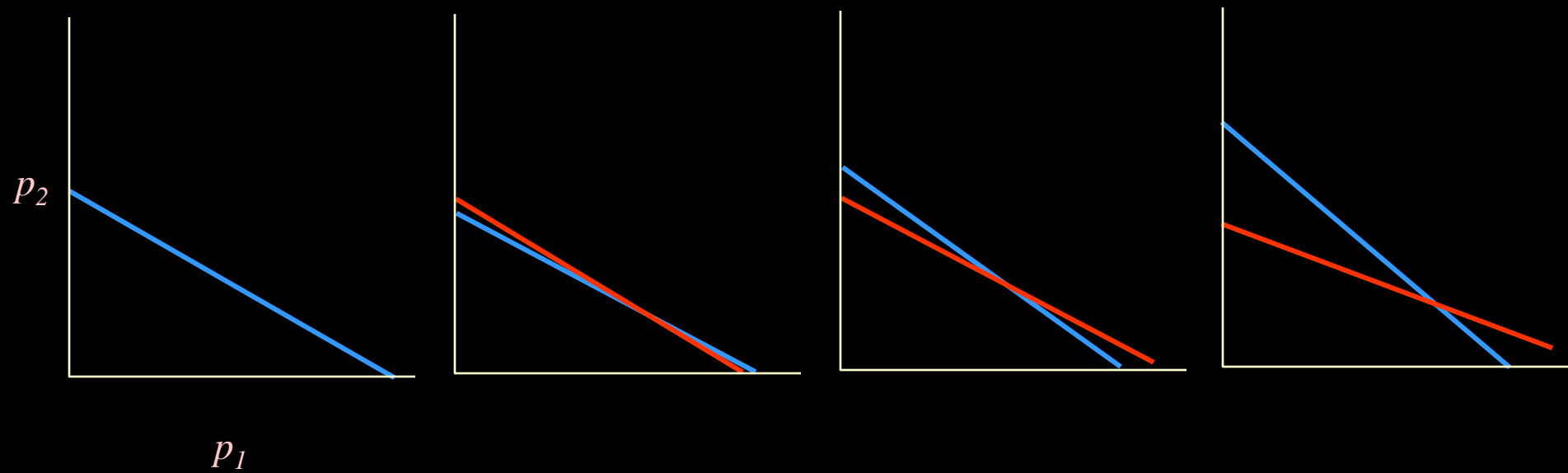
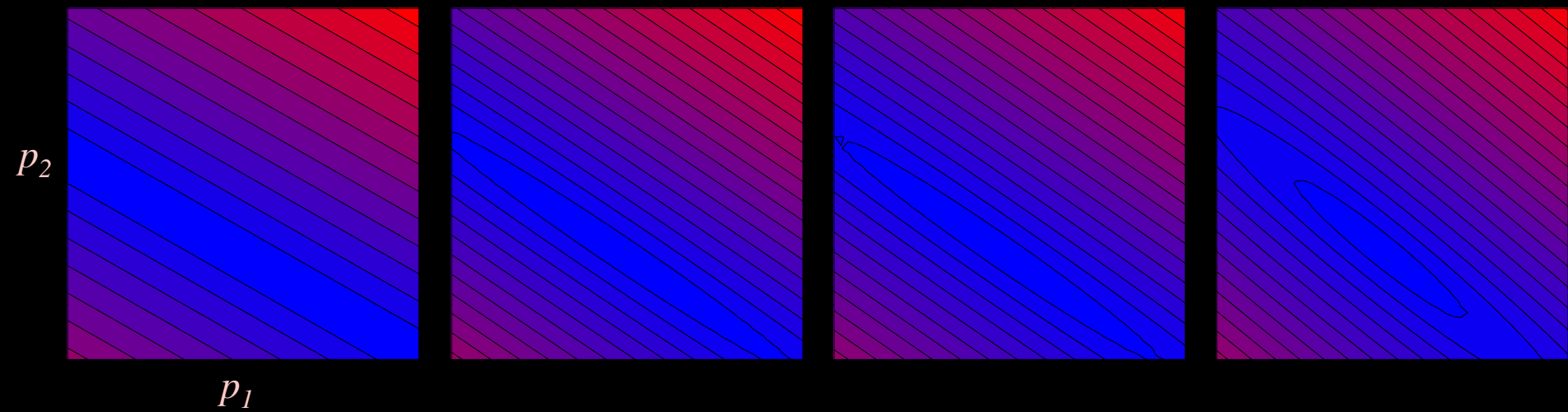
Para garantir a existência, ao invés de resolver o sistema:

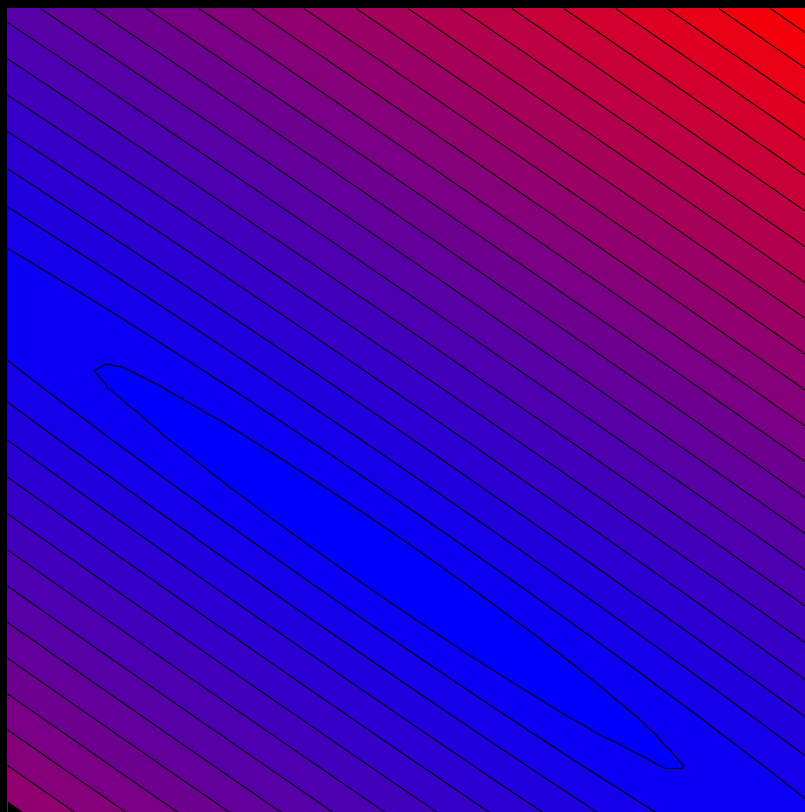
$$c_{11} p_1 + c_{12} p_2 = y_1^o$$

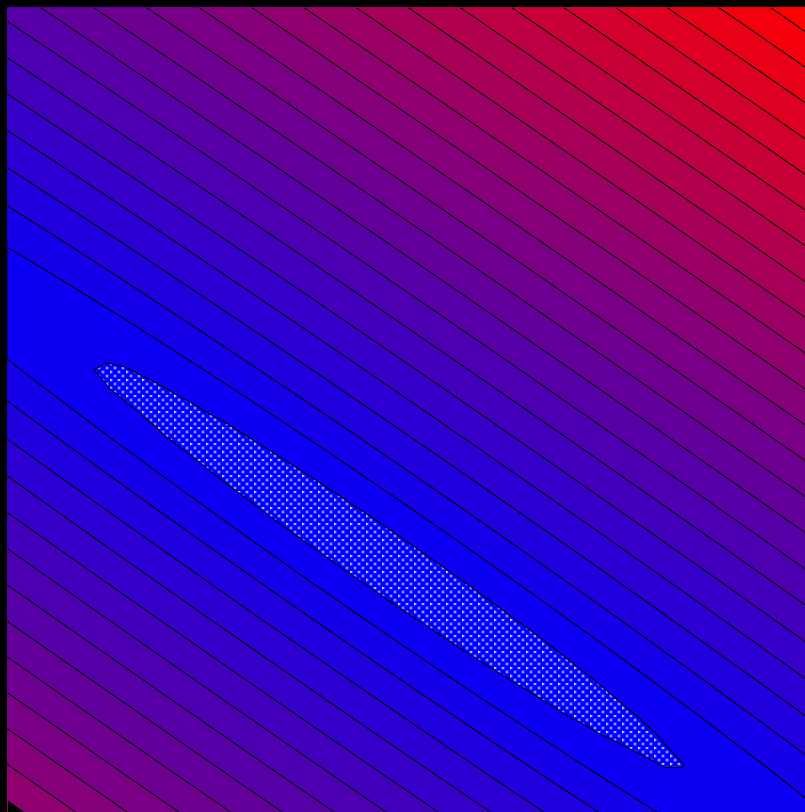
$$c_{21} p_1 + c_{22} p_2 = y_2^o$$

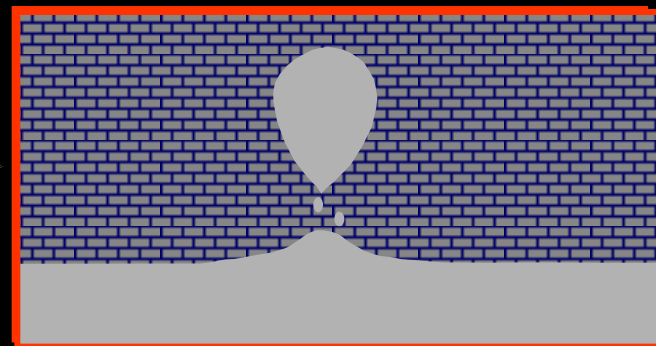
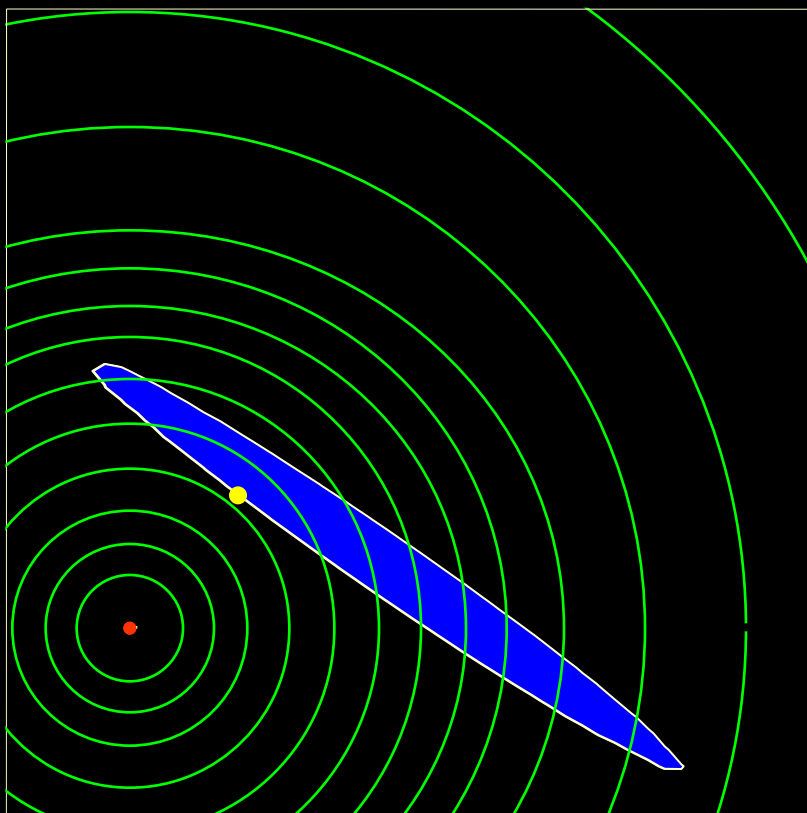
minimiza-se:

$$(y_1^o - c_{11} p_1 - c_{12} p_2)^2 + (y_2^o - c_{12} p_1 - c_{22} p_2)^2$$

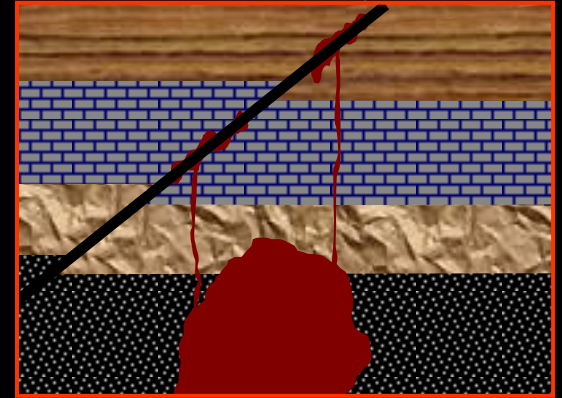
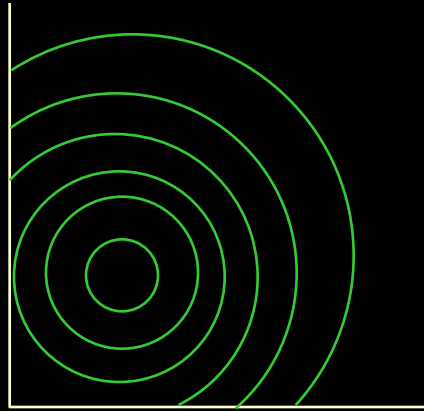






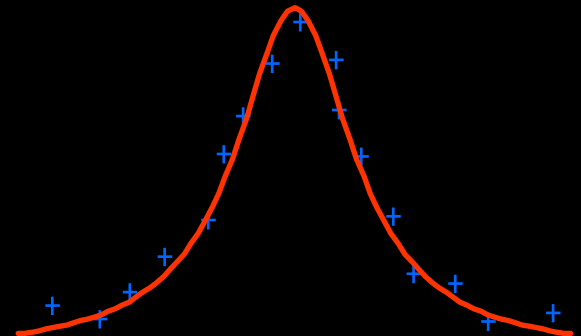
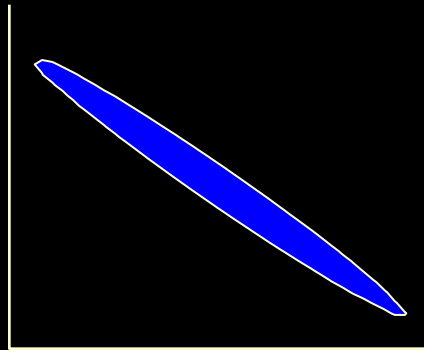


$$\min \Phi(\mathbf{p})$$



sujeito a

$$(\mathbf{y}^o - \mathbf{A}\mathbf{p})^\top (\mathbf{y}^o - \mathbf{A}\mathbf{p}) = \delta$$





# Solução via multiplicadores de Lagrange

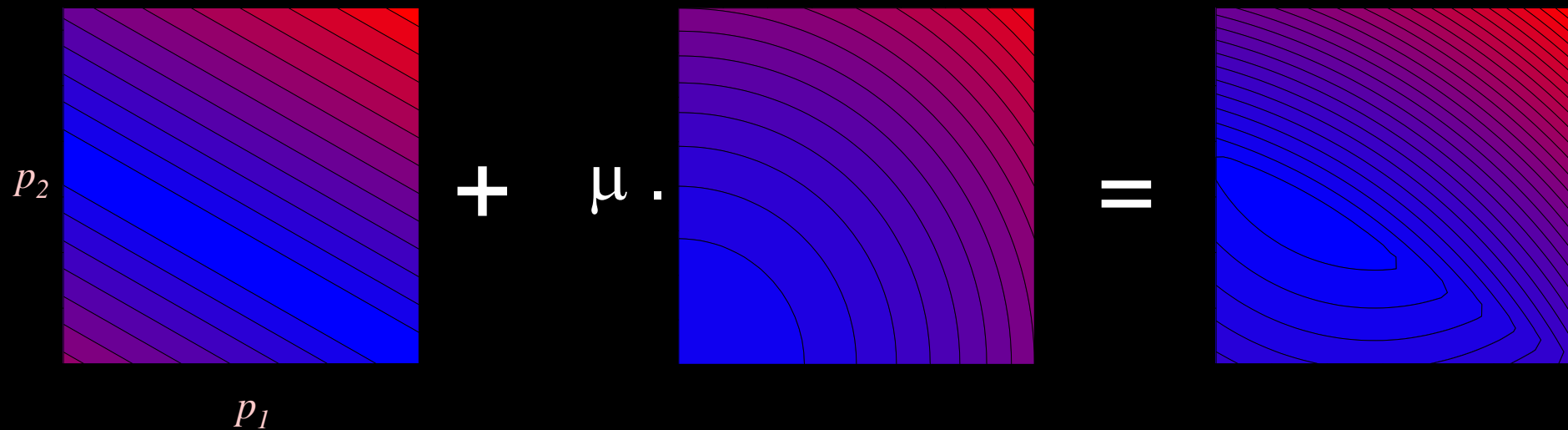
The diagram illustrates the process of solving a constrained minimization problem using Lagrange multipliers. It starts with a minimization problem with a constraint, which is then transformed into an unconstrained minimization problem by introducing a Lagrange multiplier  $\mu$ .

$$\left\{ \begin{array}{l} \min \Phi(\mathbf{p}) \\ \text{sujeito a} \\ (\mathbf{y}^o - \mathbf{A}\mathbf{p})^\top (\mathbf{y}^o - \mathbf{A}\mathbf{p}) = \delta \end{array} \right.$$
$$\min \tau = (\mathbf{y}^o - \mathbf{A}\mathbf{p})^\top (\mathbf{y}^o - \mathbf{A}\mathbf{p}) + \mu \Phi(\mathbf{p})$$

$$(\mathbf{y}^\circ - \mathbf{A}\mathbf{p})^\top (\mathbf{y}^\circ - \mathbf{A}\mathbf{p})$$

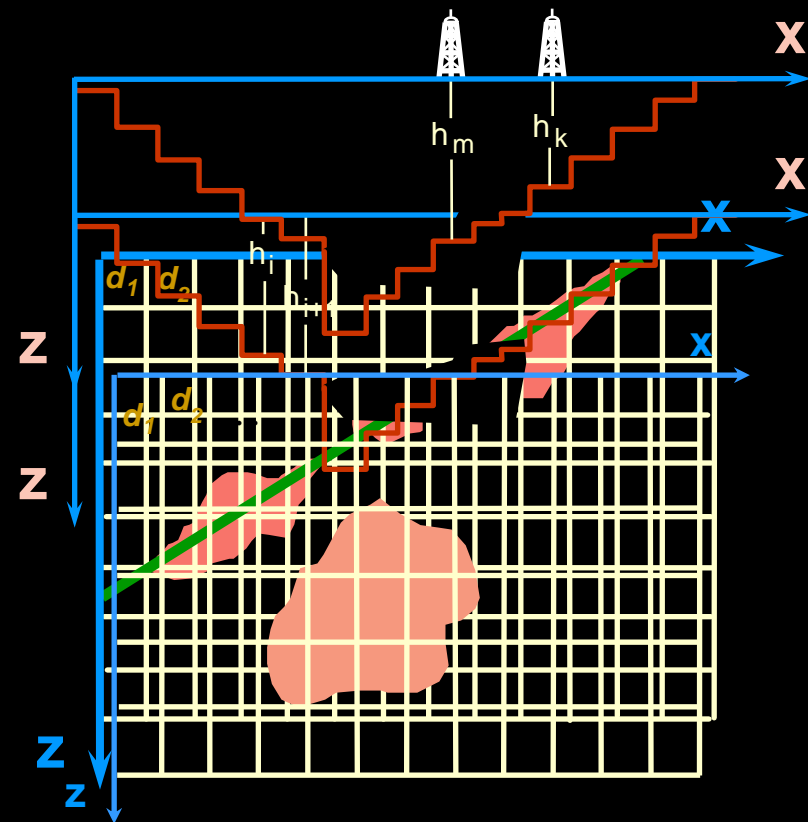
$\equiv$

$$\|\mathbf{y}^\circ - \mathbf{A}\mathbf{p}\|_2 + \mu \cdot \Phi(\mathbf{p}) = \tau$$



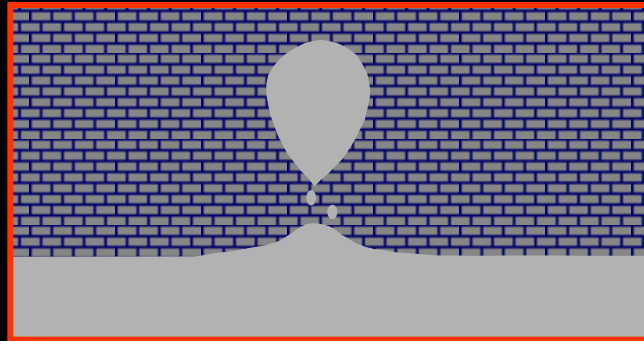
# Funcionais estabilizadores: (Tikhonov)

- Igualdade absoluta
- Igualdade relativa
- Mínimo momento de inércia
- Compacidade

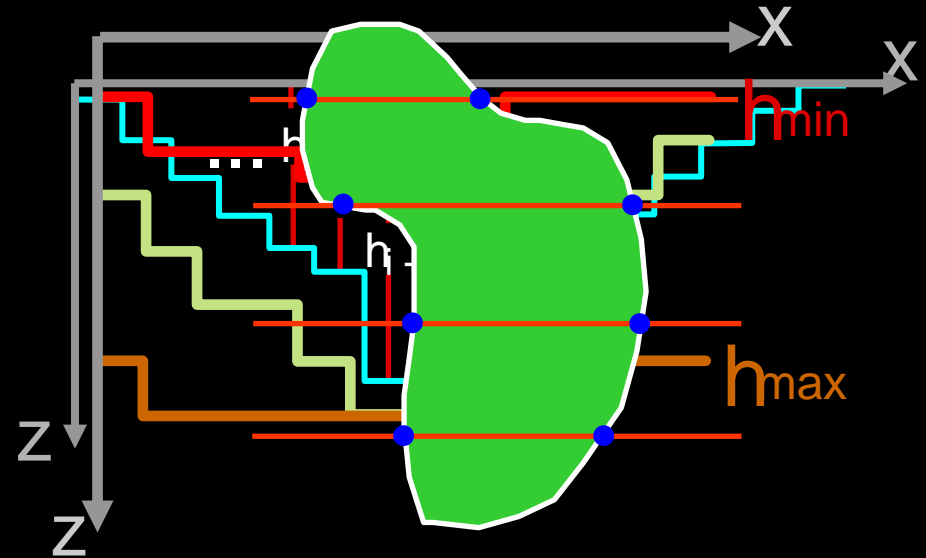


# Funcionais estabilizadores:

- Convexidade global

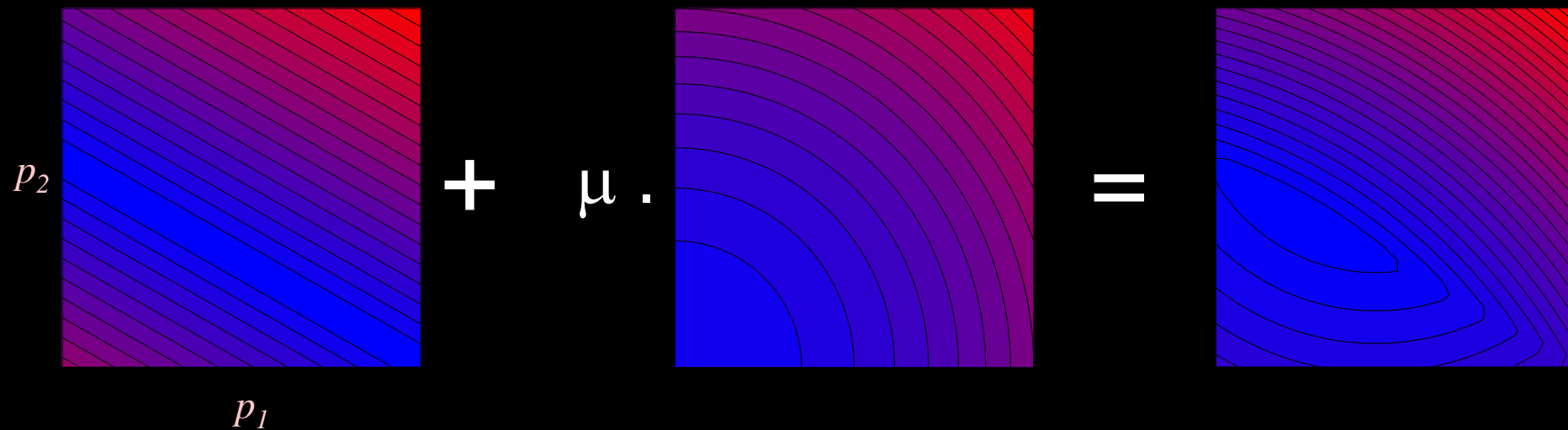


# Funcionais não estabilizadores:

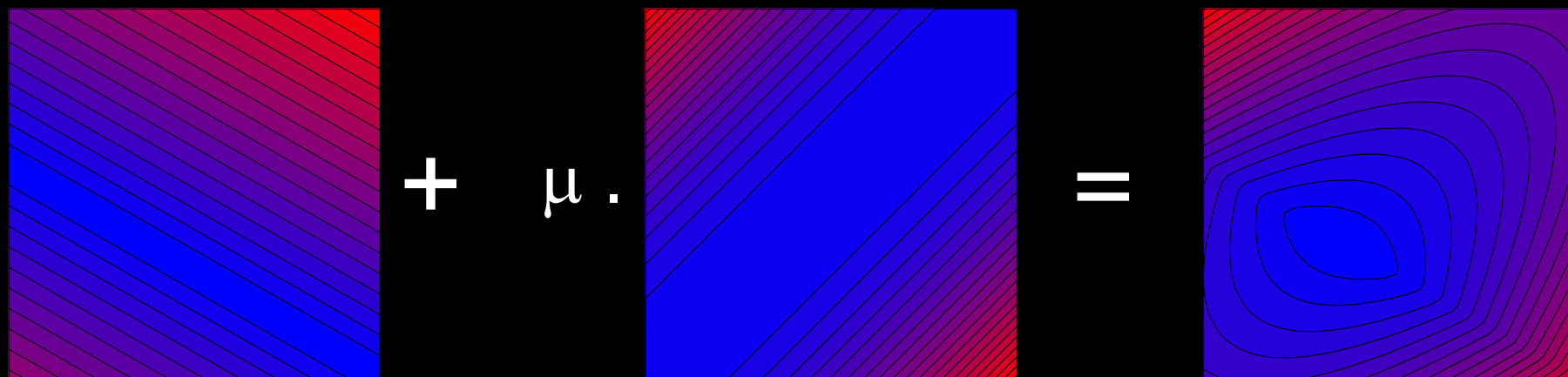


- Desigualdade
- Igualdade relativa ponderada
- ~~Convexidade direcional~~

# Igualdade absoluta



# Igualdade relativa



## Vínculos de igualdade absoluta:

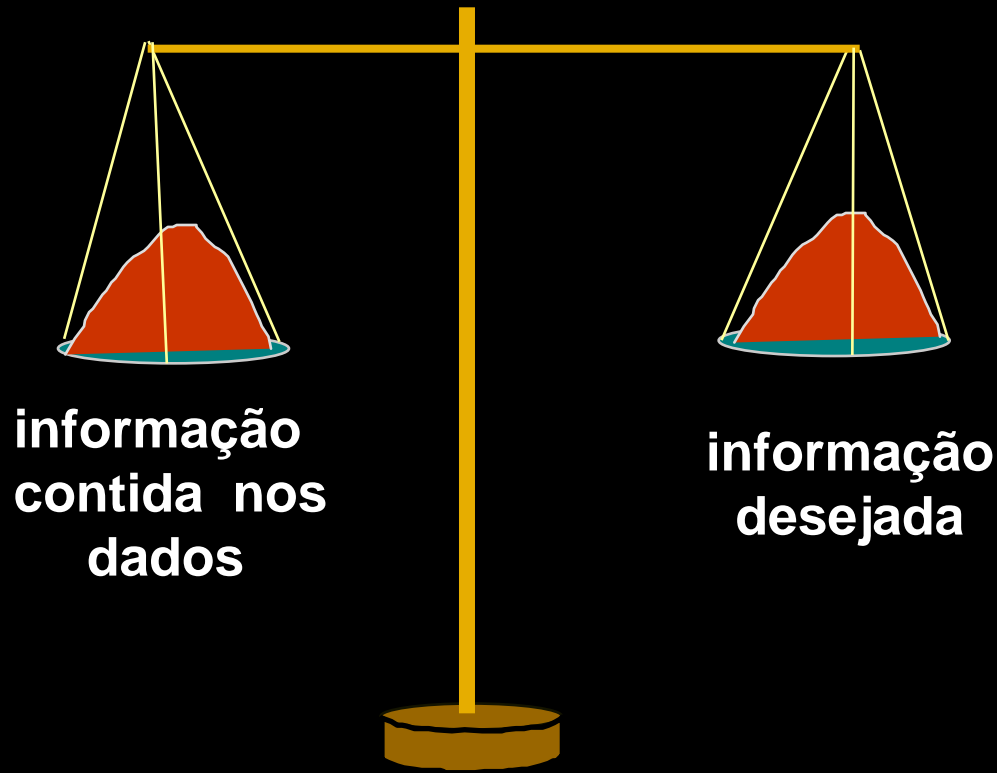
- Proximidade de todos os parâmetros a zero  
“RIDGE REGRESSION”

## Vínculos de igualdade relativa:

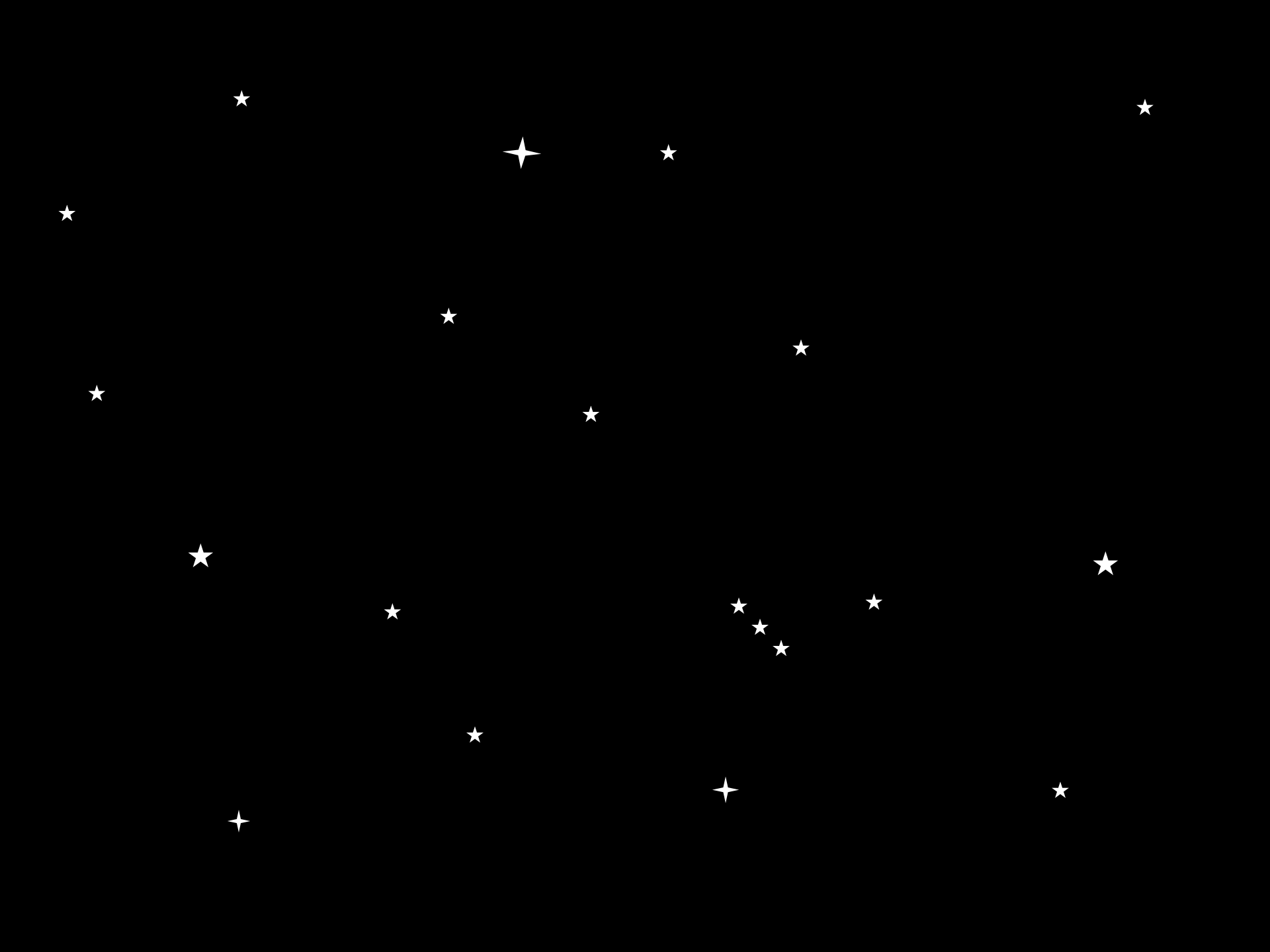
- Proximidade entre parâmetros espacialmente adjacentes  
“SUAVIDADE”

# Tikhonov:

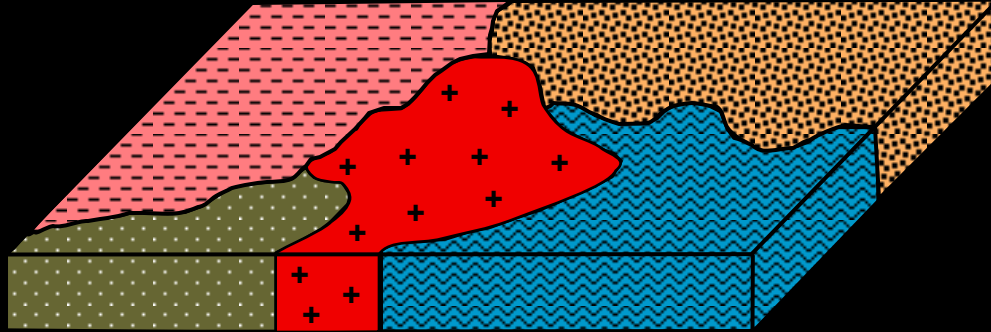
Introduzir informação a priori







# Mapeamento de densidade ou susceptibilidade

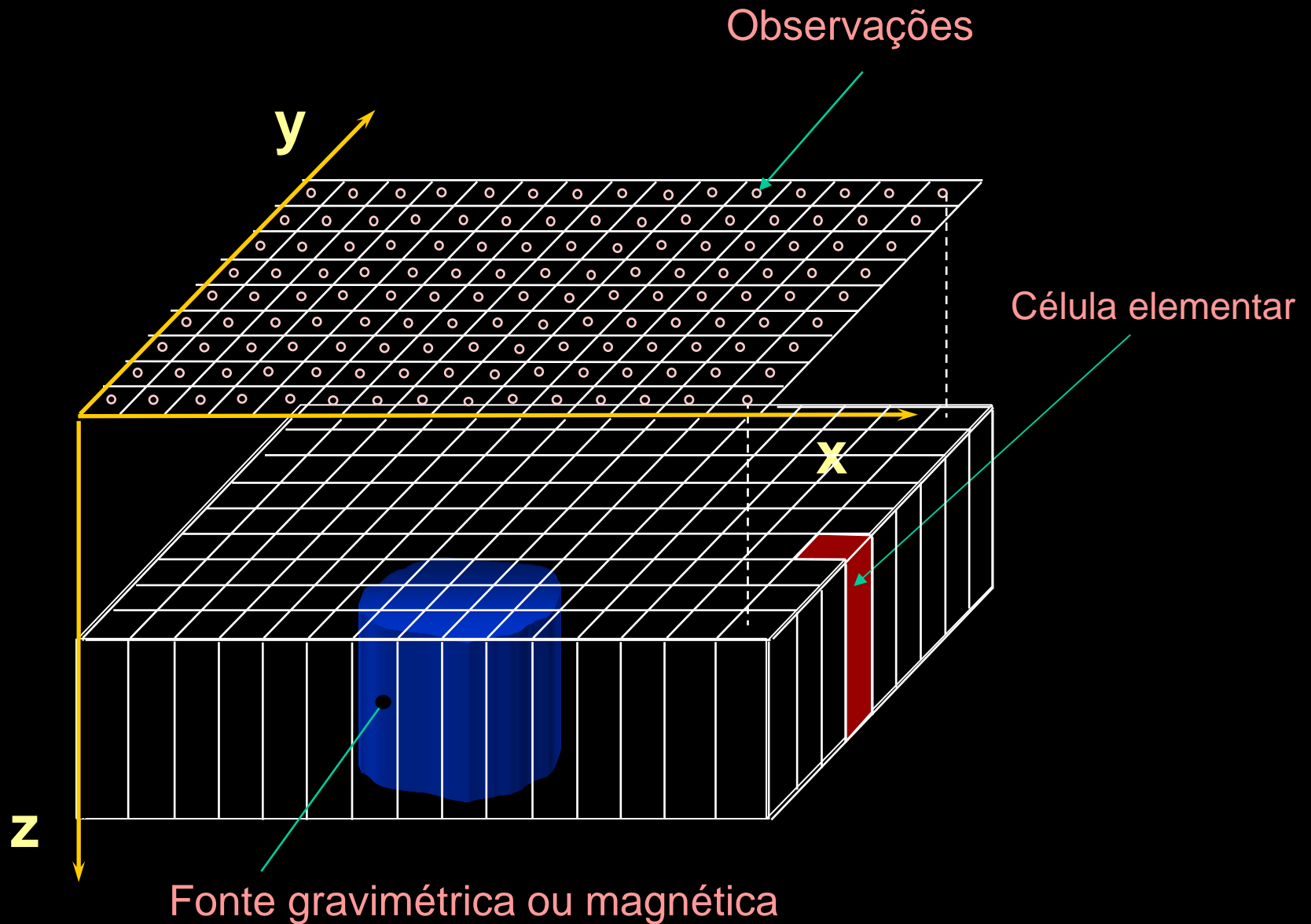


**Problema geológico:** mapeamento em planta de diversas unidades geológicas

**Simplificações:**

- a) formações com espessuras finitas
- b) topo e base planos, horizontais e conhecidos
- c) contatos verticais

# Modelo interpretativo



# Problema matemático:

$$A p = y^o$$

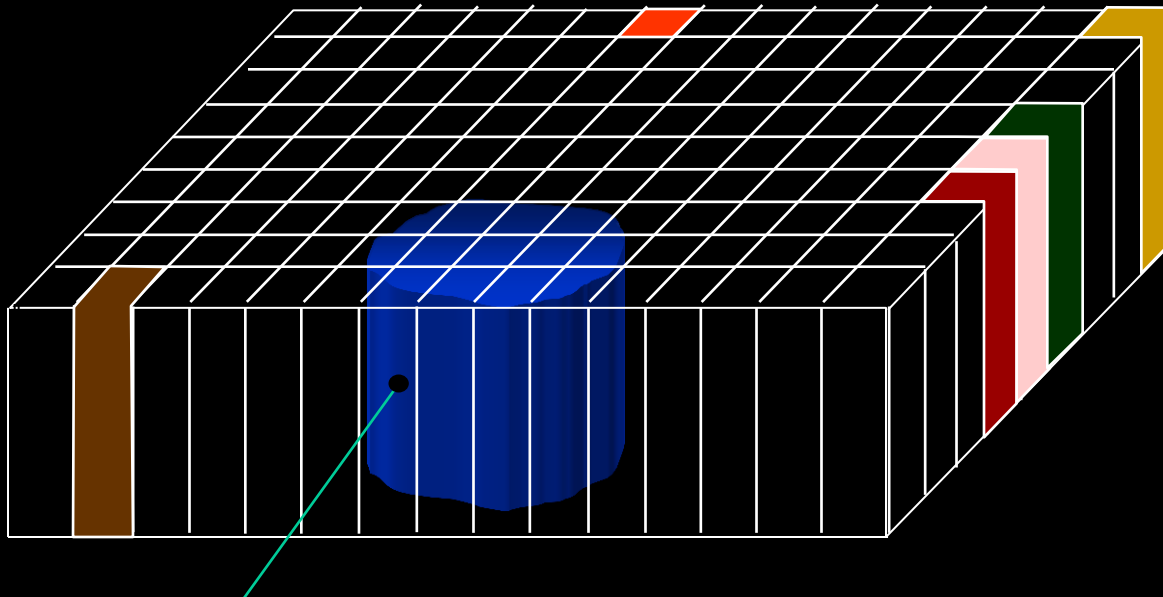
Para garantir existência de uma solução,  
minimiza-se:

$$(y^o - Ap)^T (y^o - Ap)$$

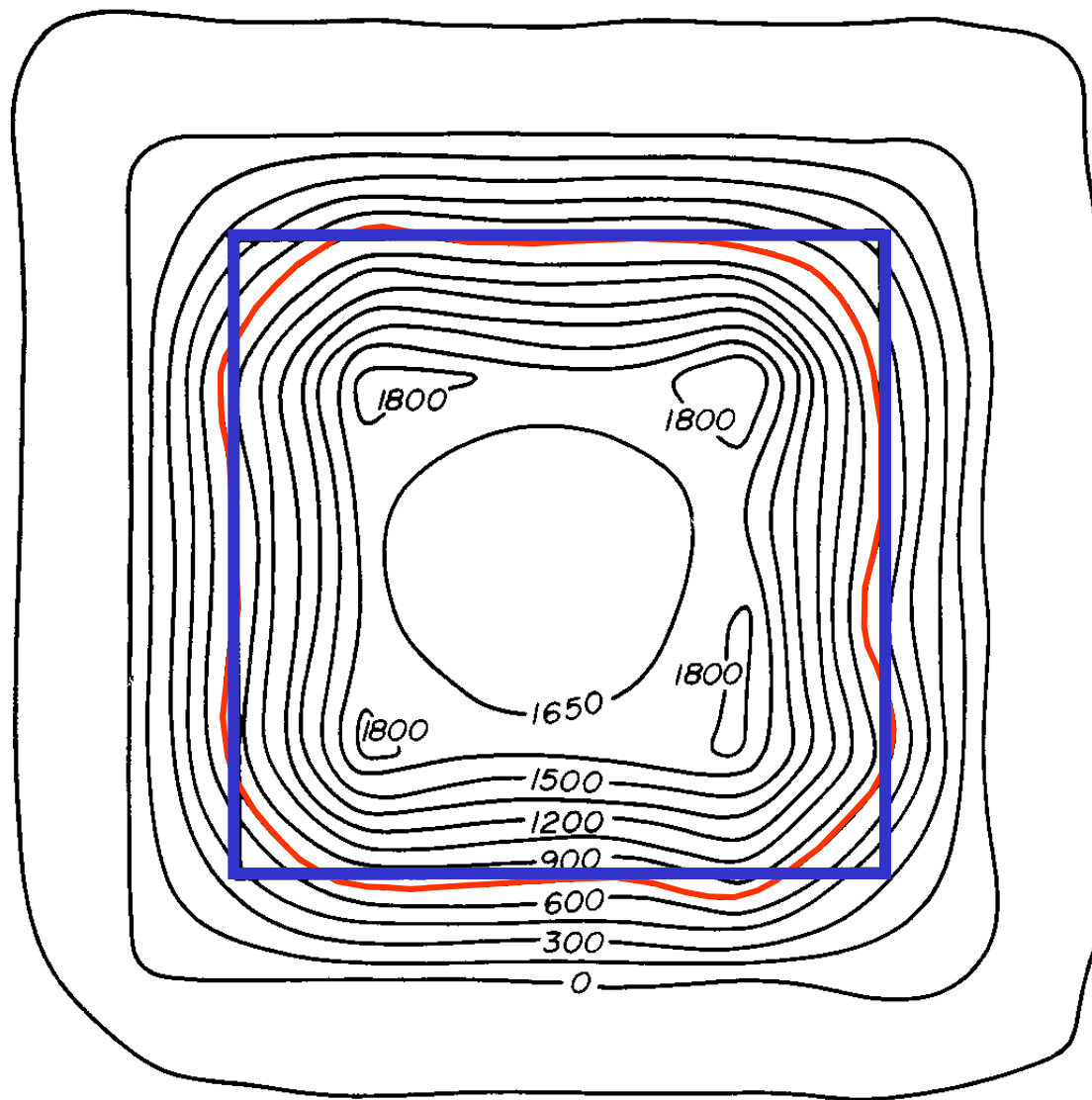
# Estabilizadores:

**Ridge:** proximidade de todos os parâmetros de zero

**Suavidade:** proximidade entre todos os parâmetros adjacentes

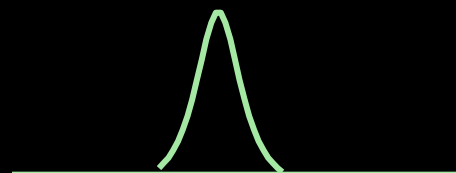
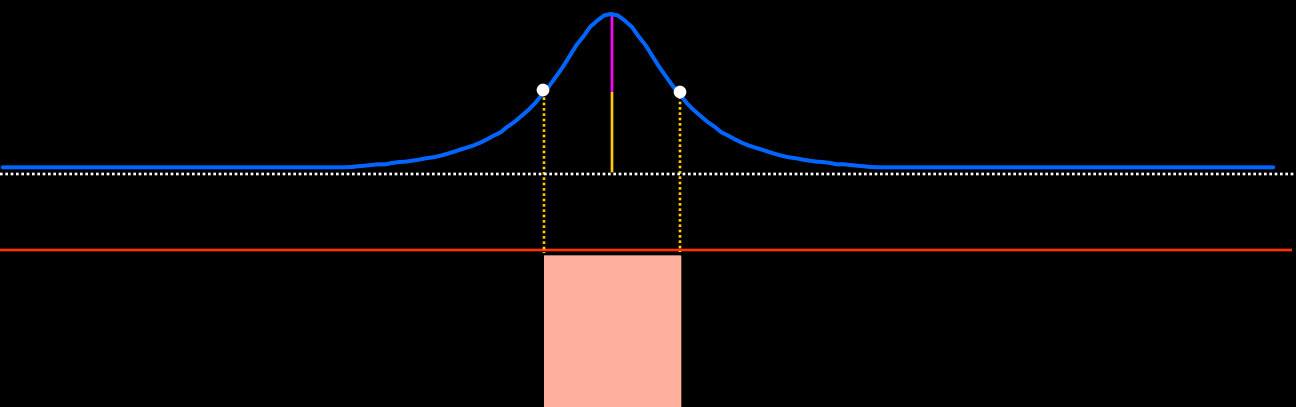


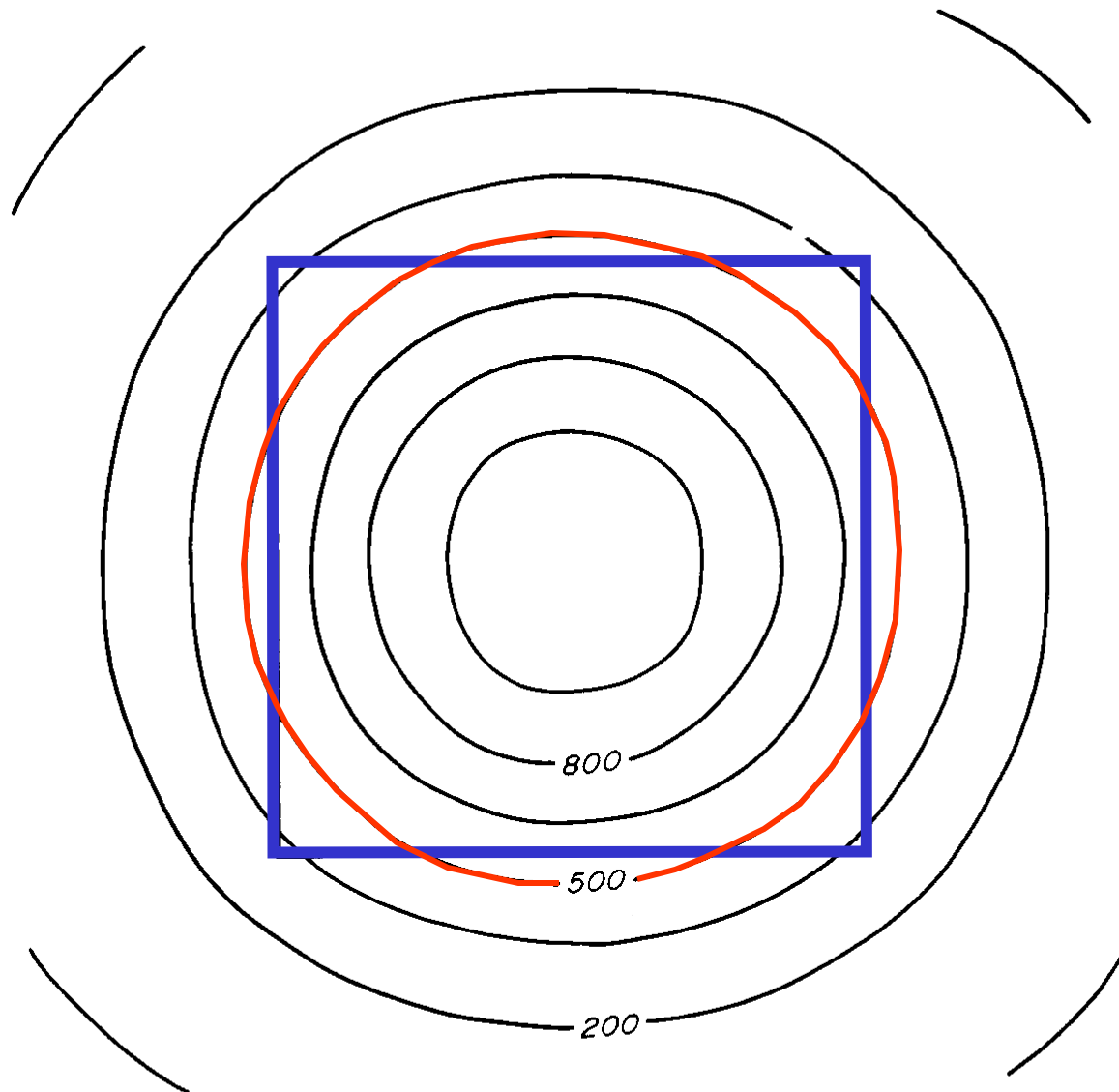
Fonte gravimétrica ou magnética



GRID UNIT



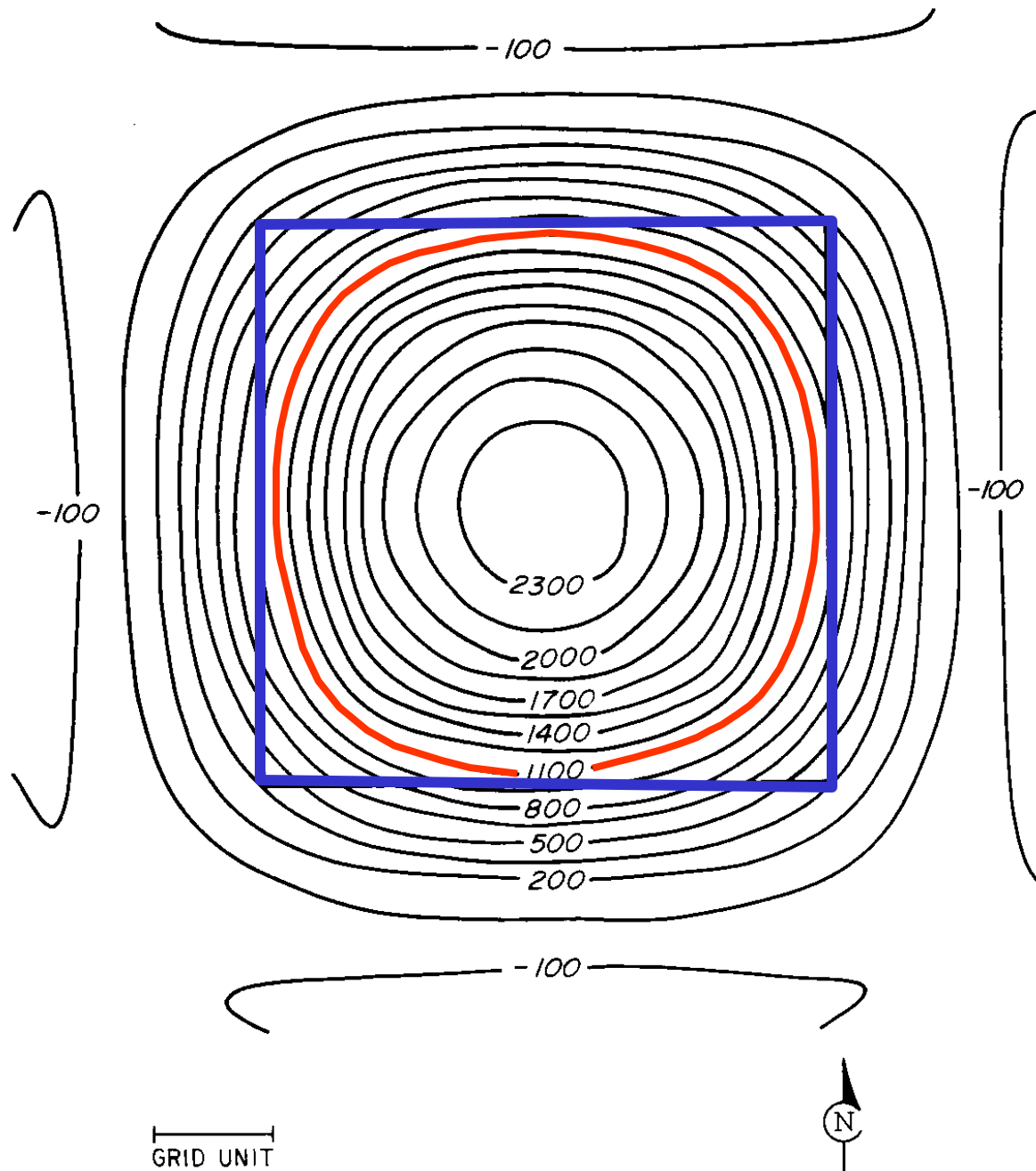


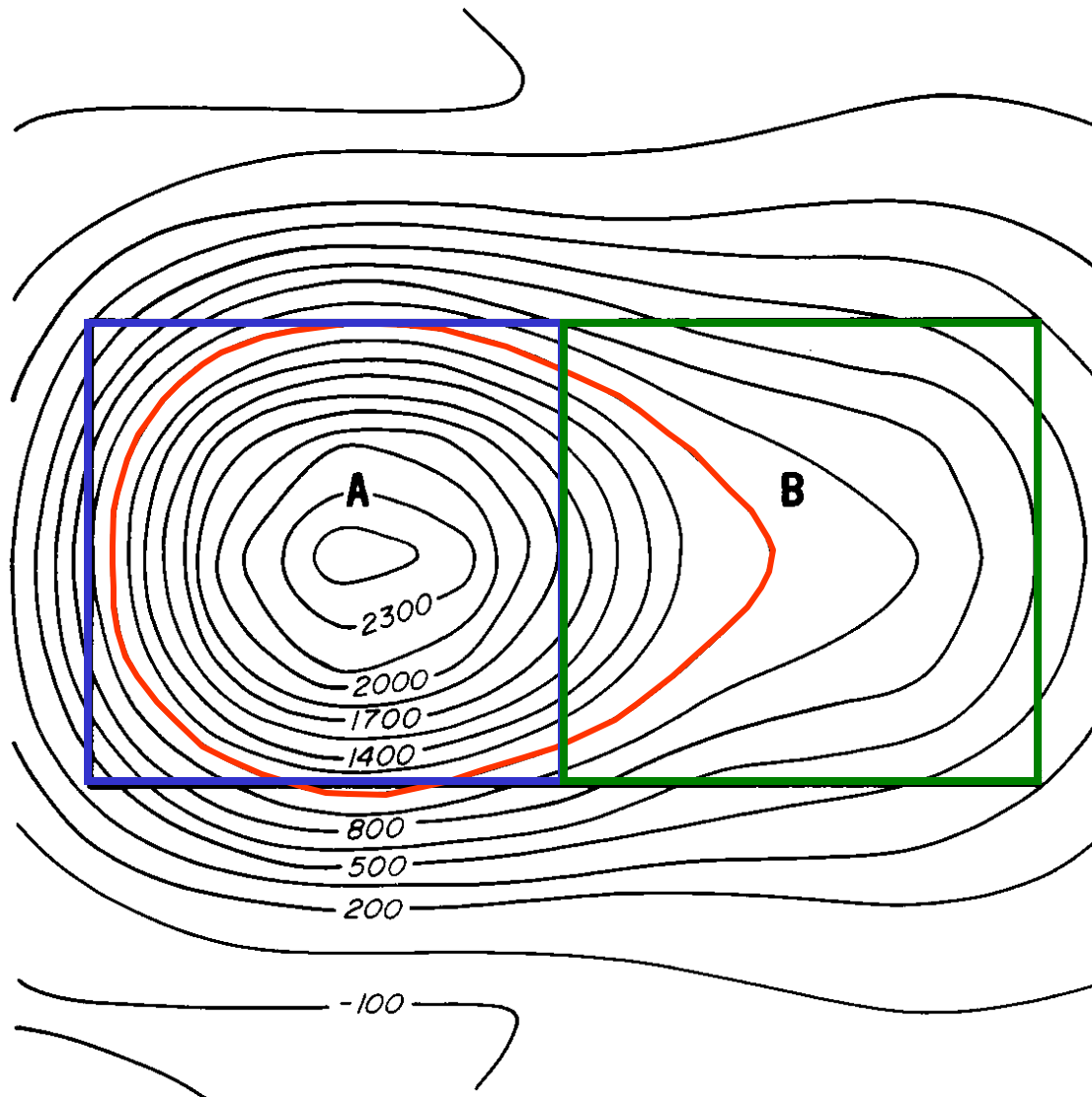


GRID UNIT



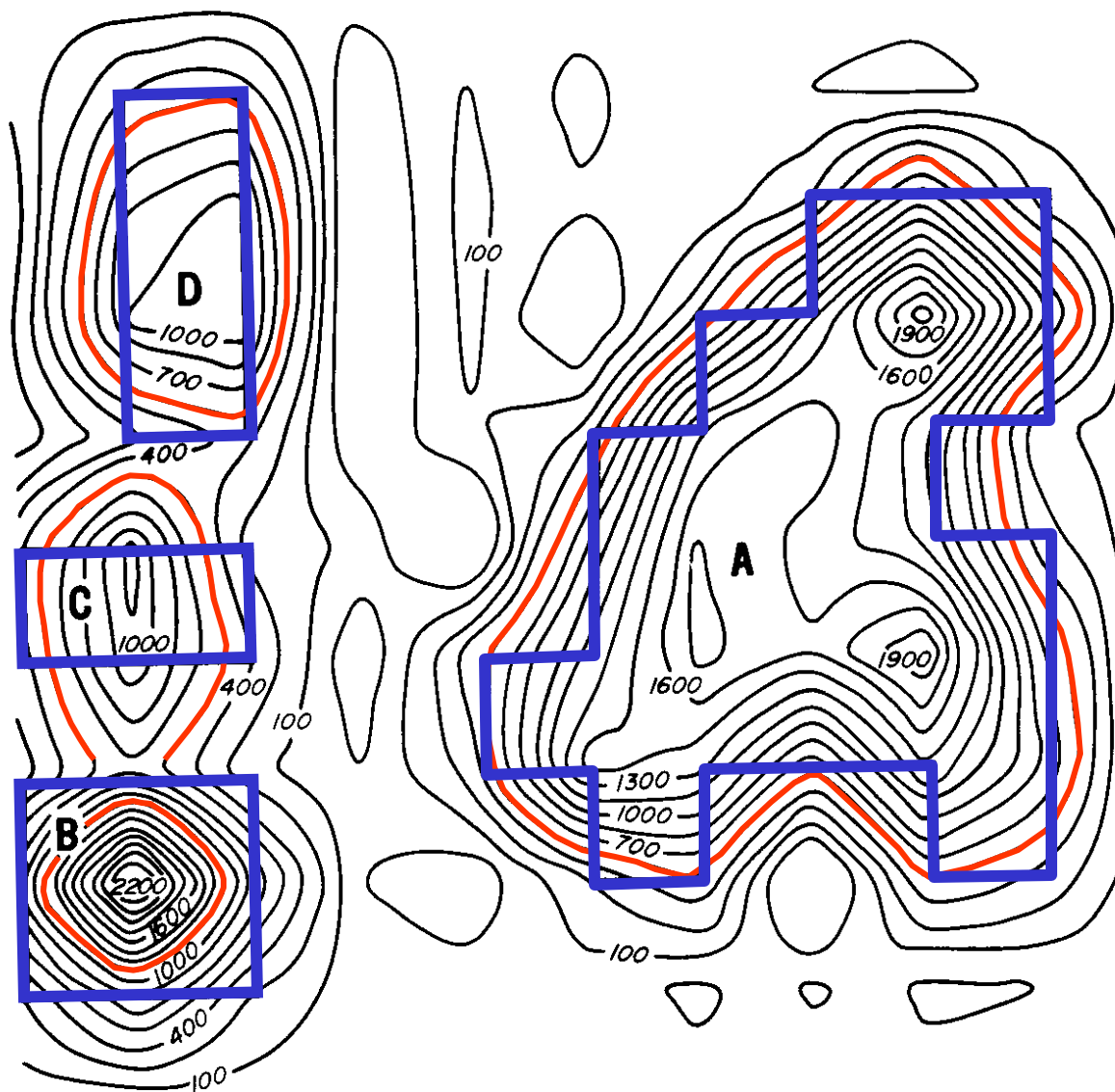






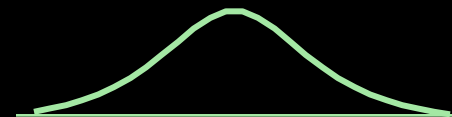
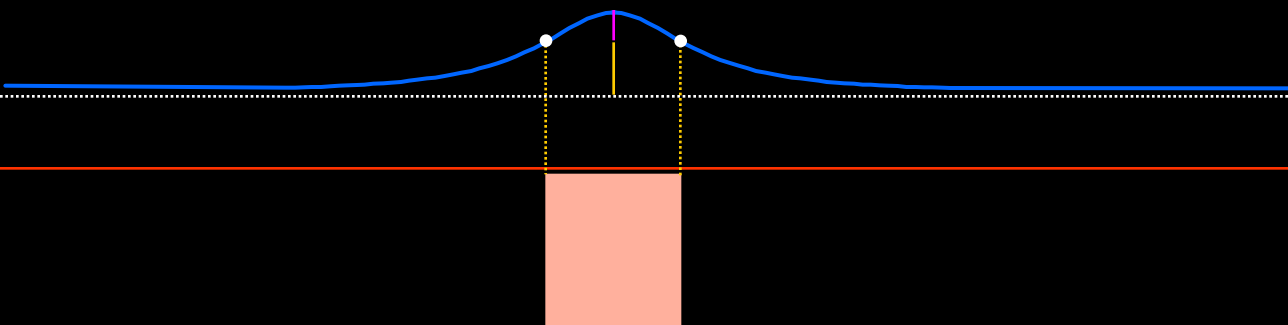
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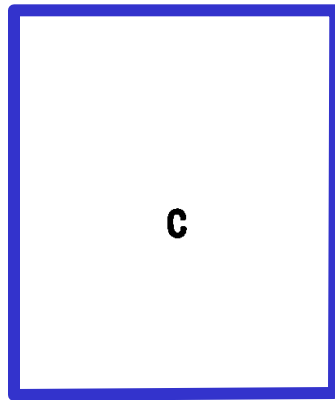




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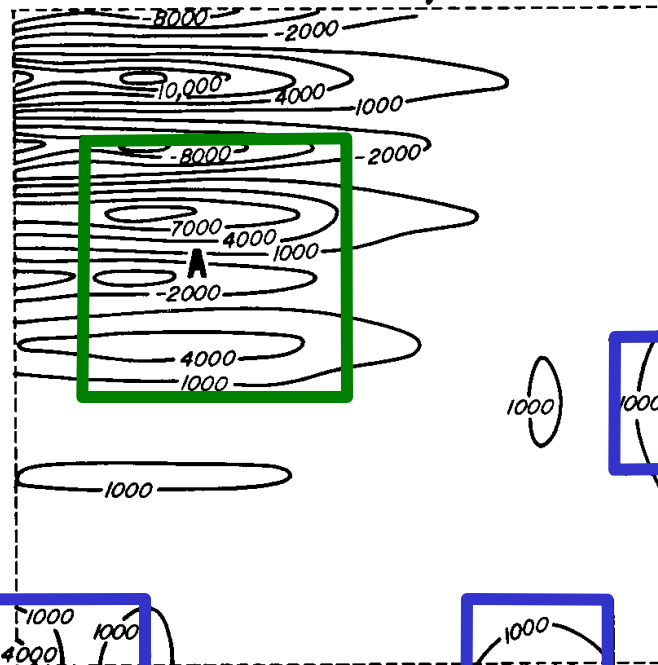






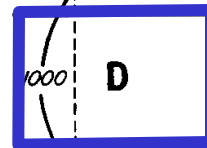
**C**

*Grid*

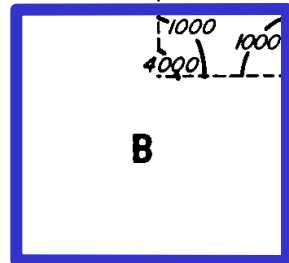


**A**

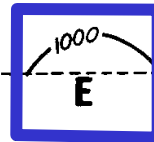
1000



**D**

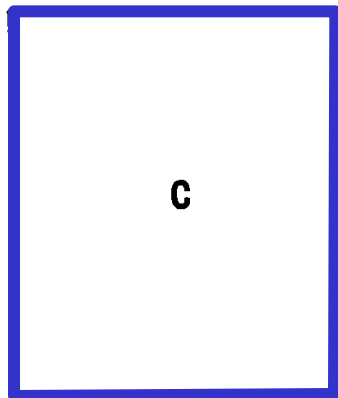


**B**

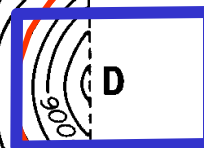
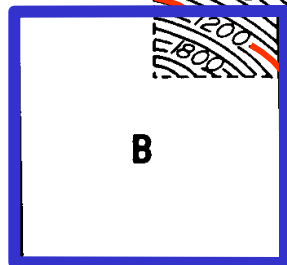
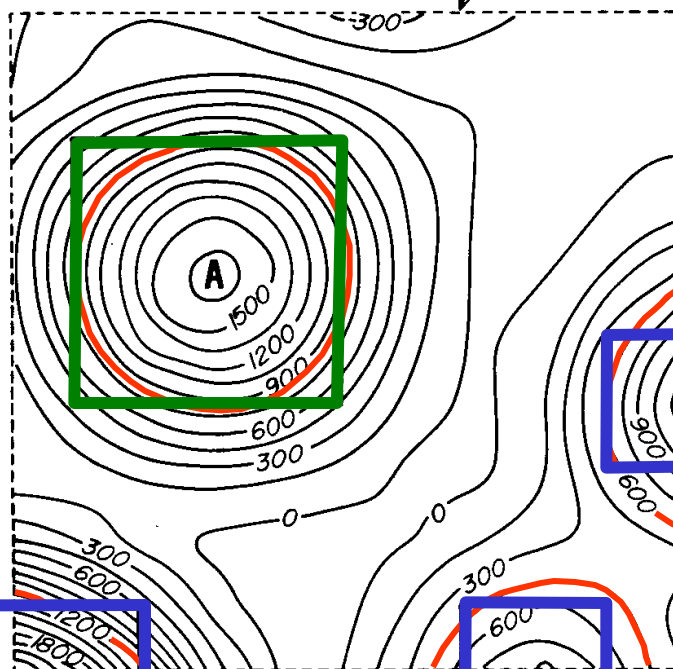


**E**

GRID UNIT

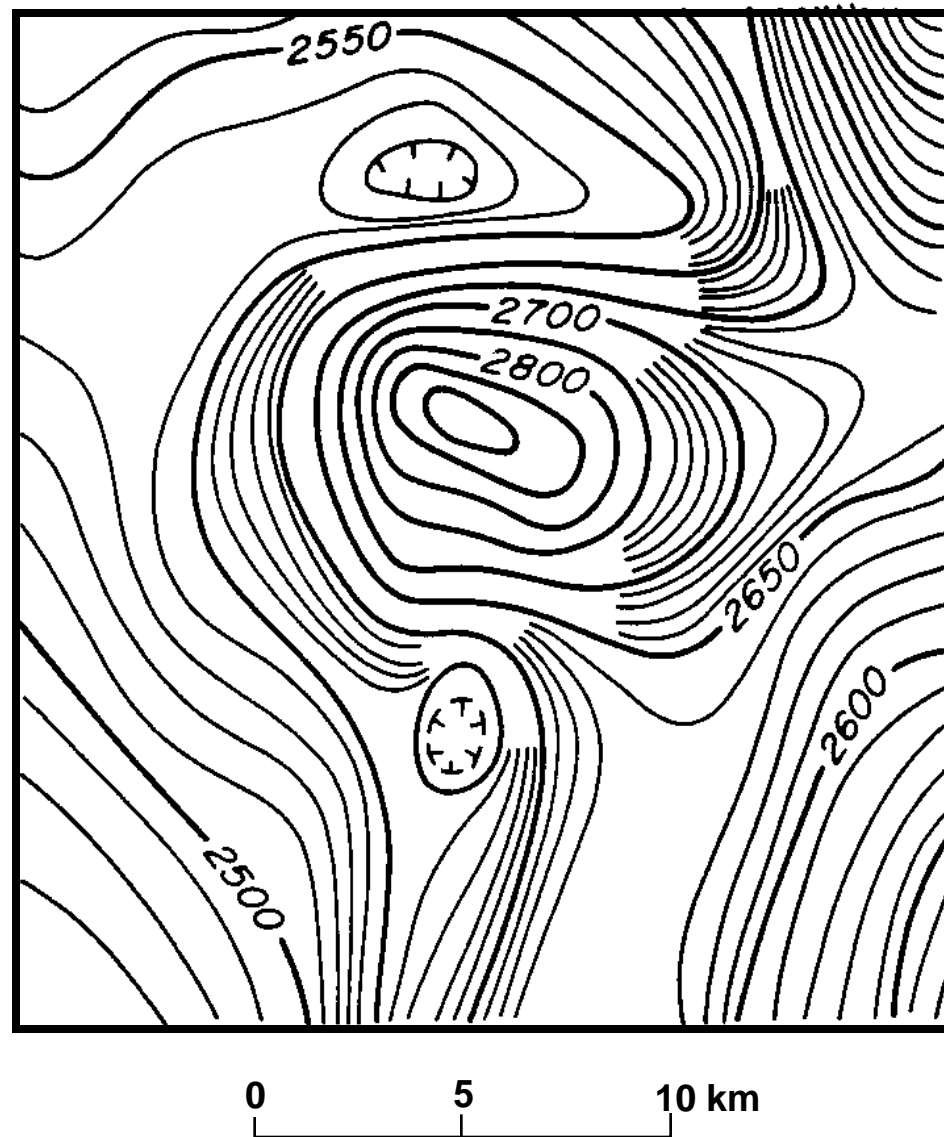


Grid

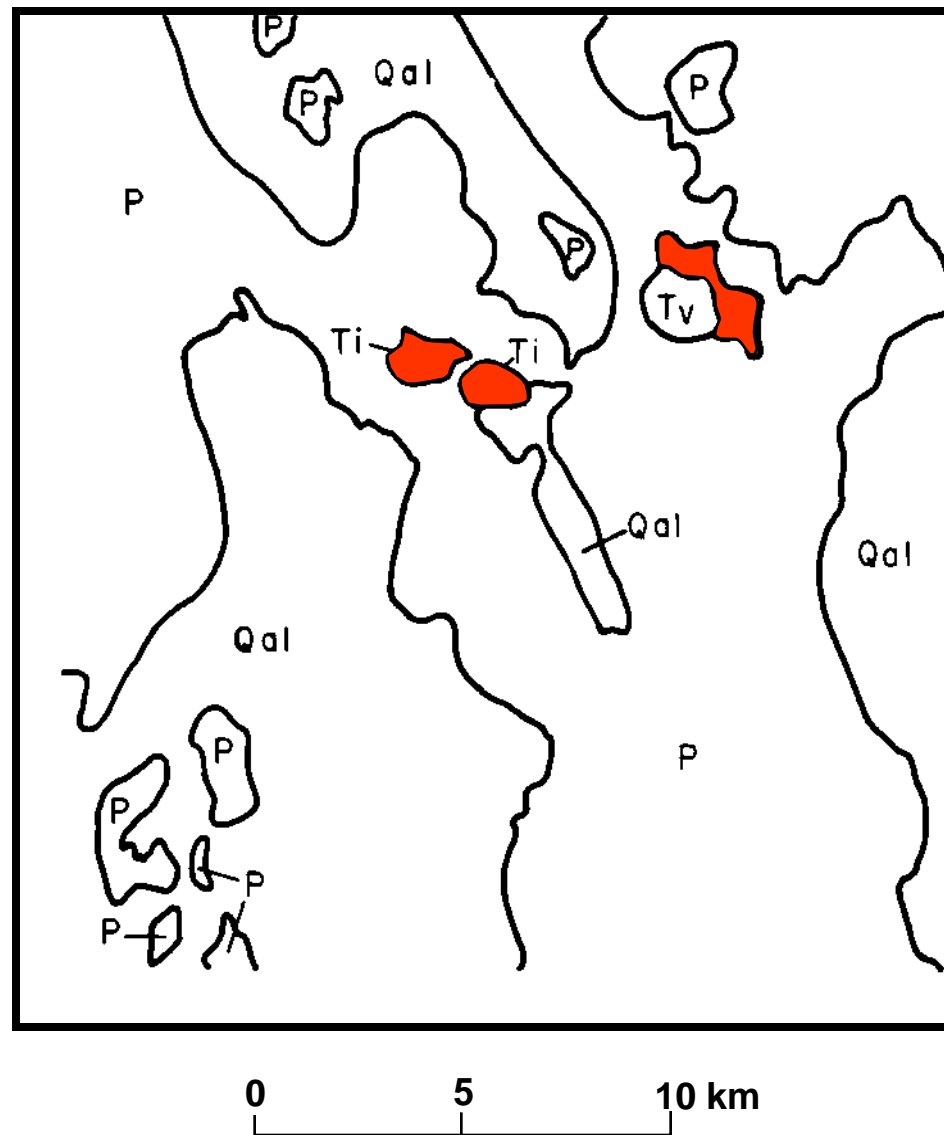


GRID UNIT

# Ely, Nevada

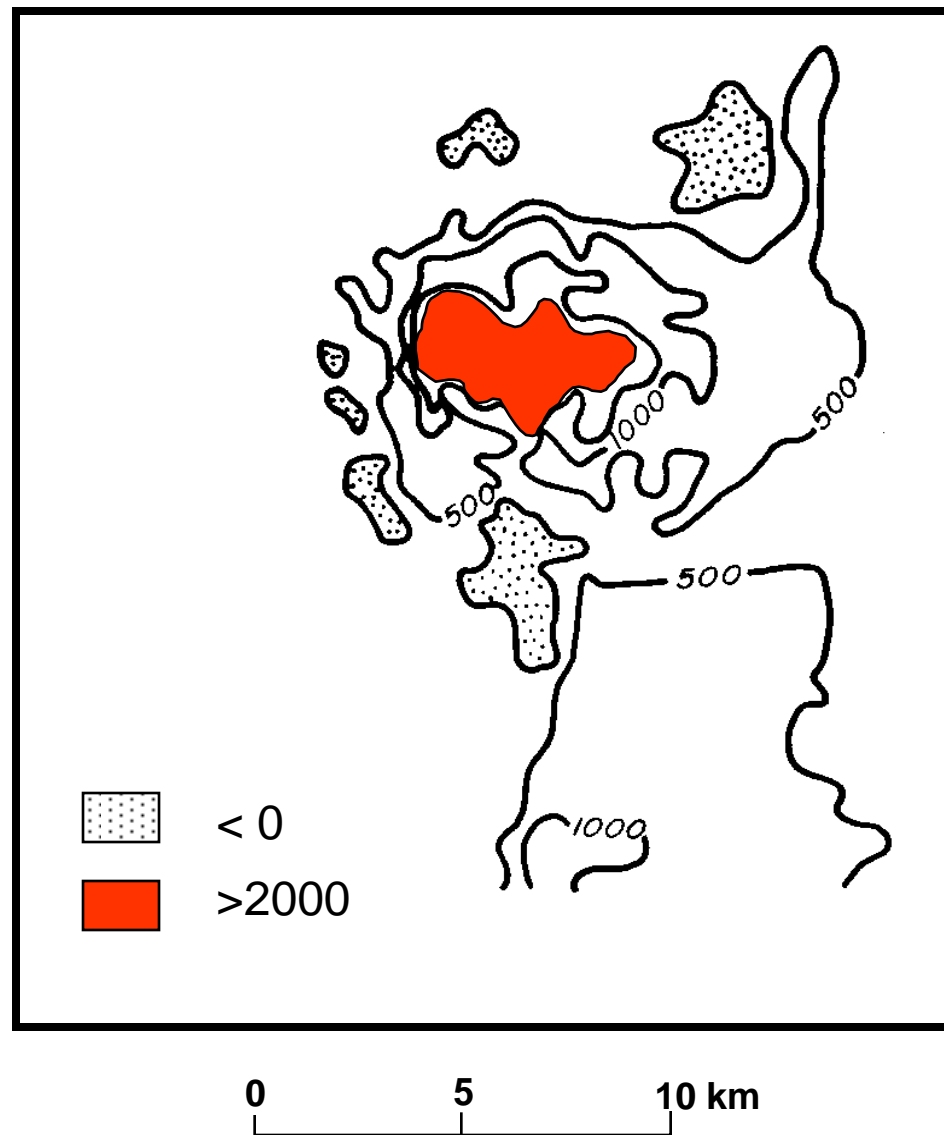


## Ely, Nevada

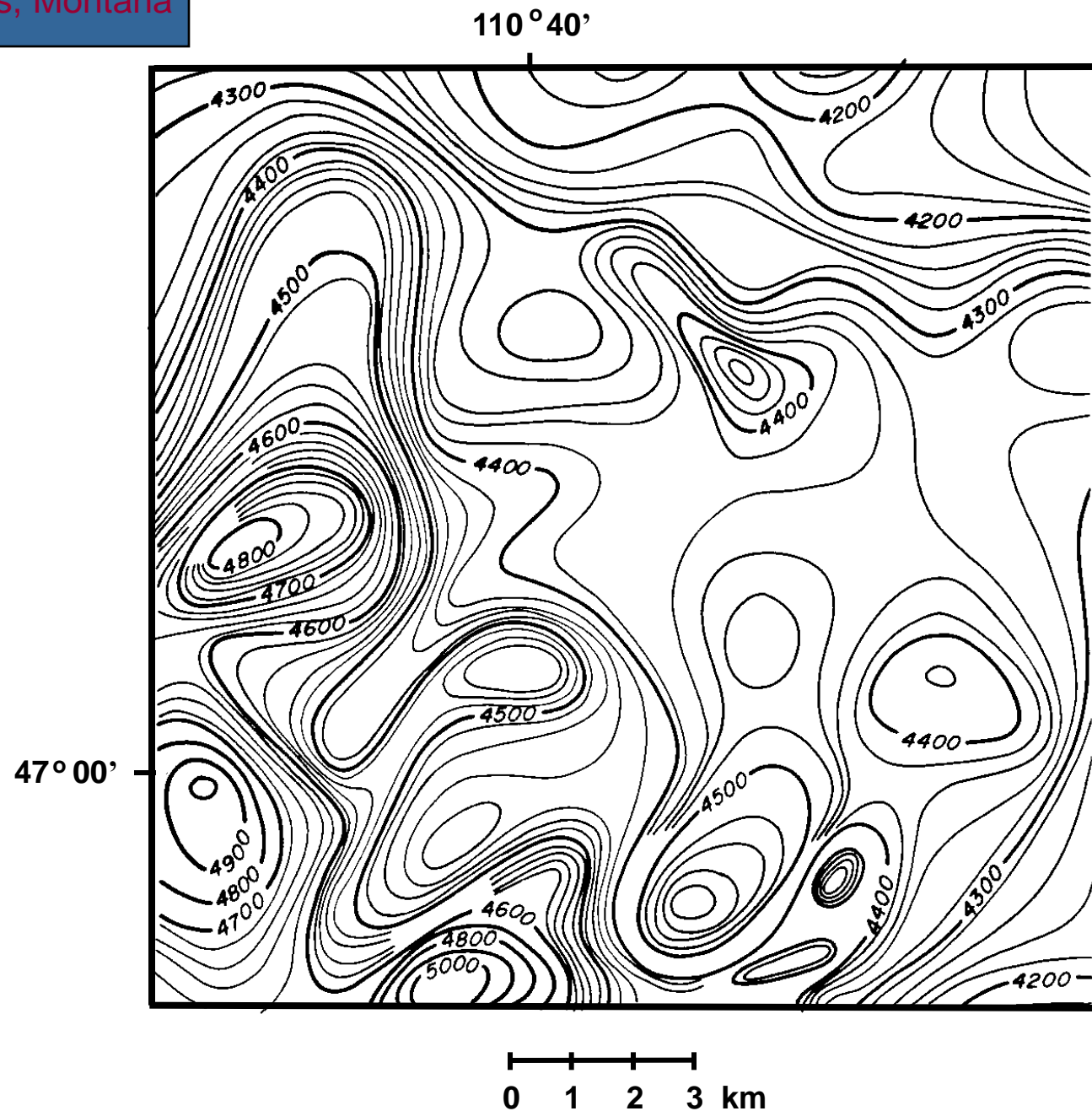




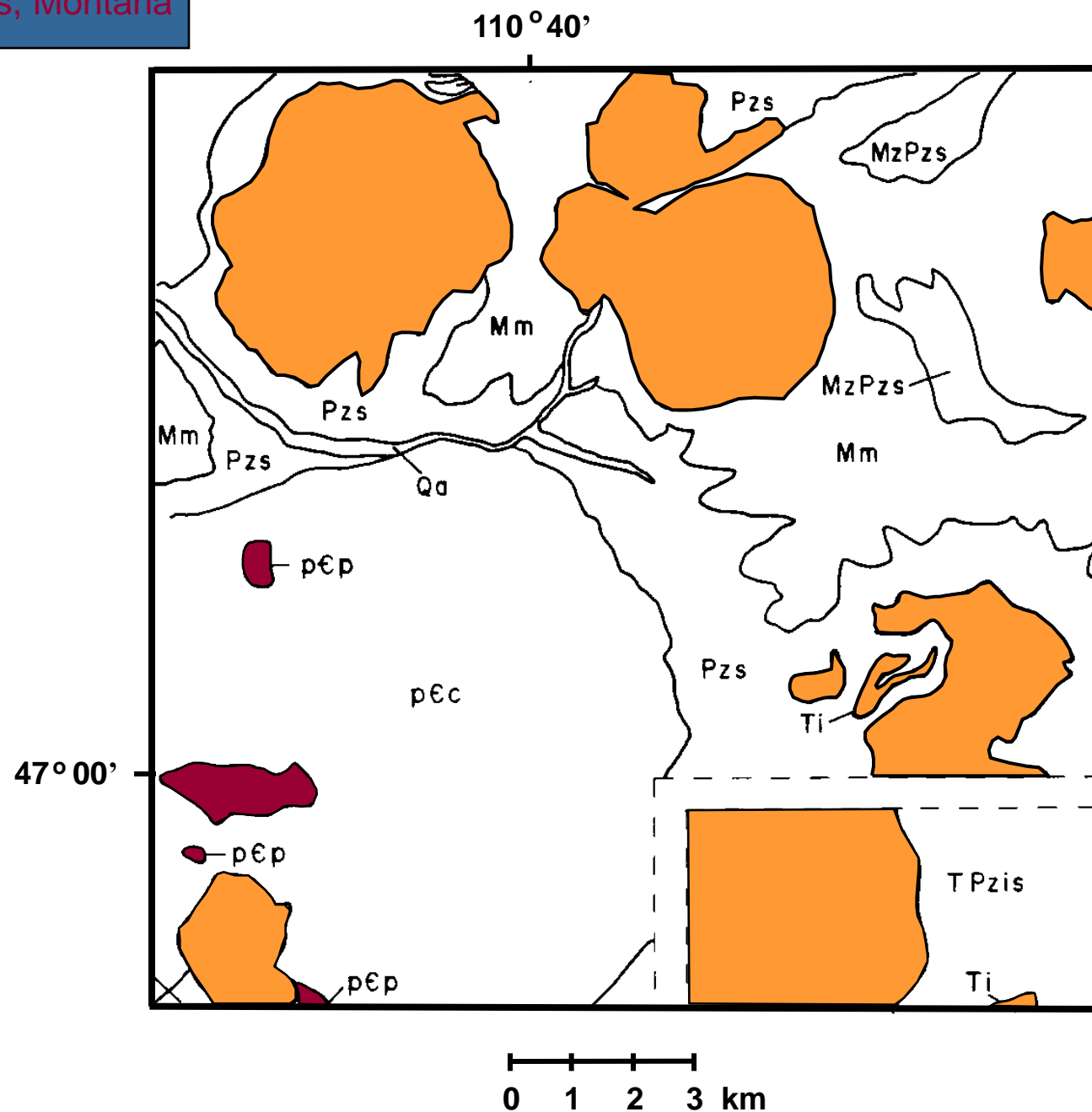
# Ely, Nevada



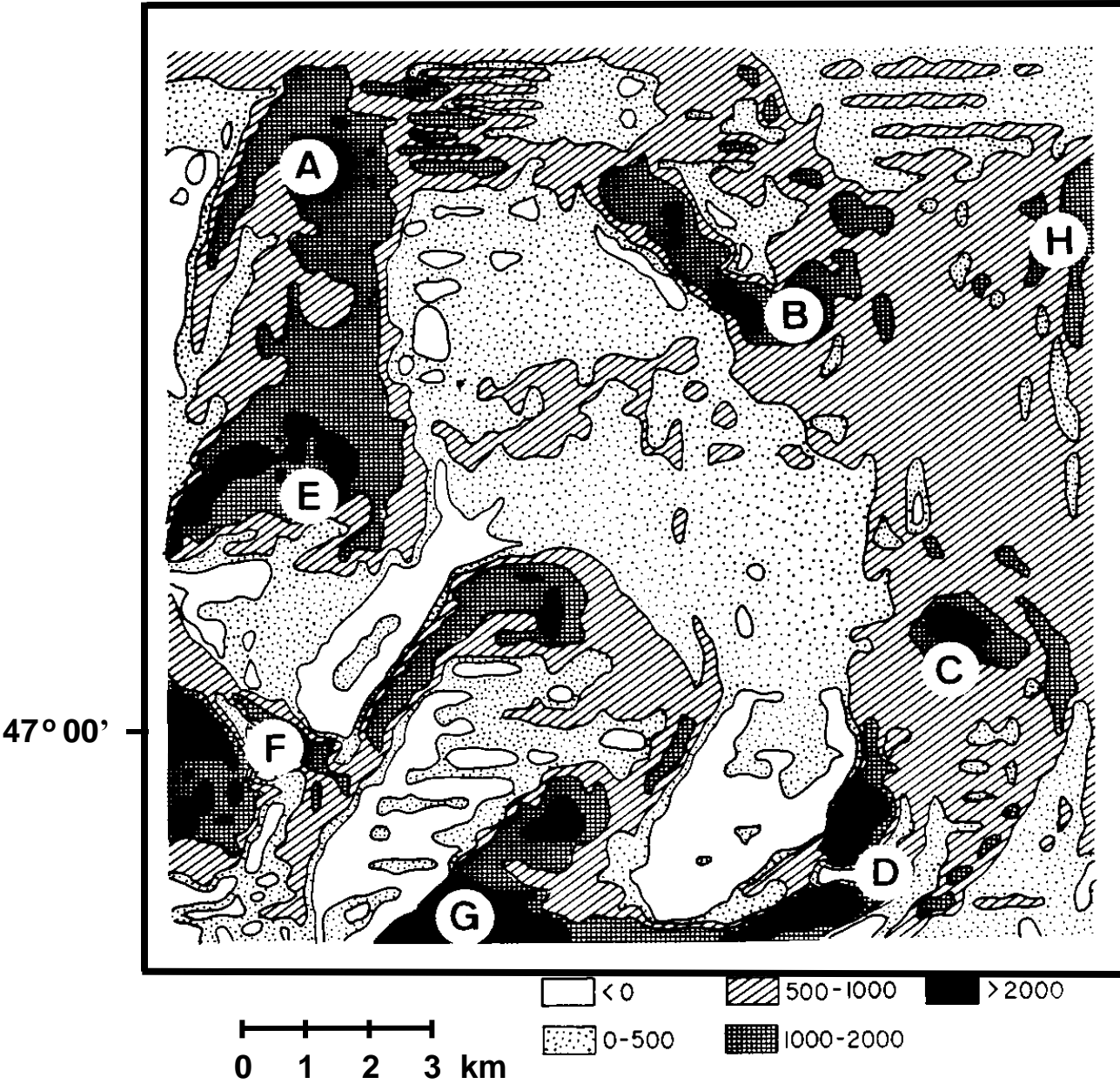
Little Belt Mts, Montana

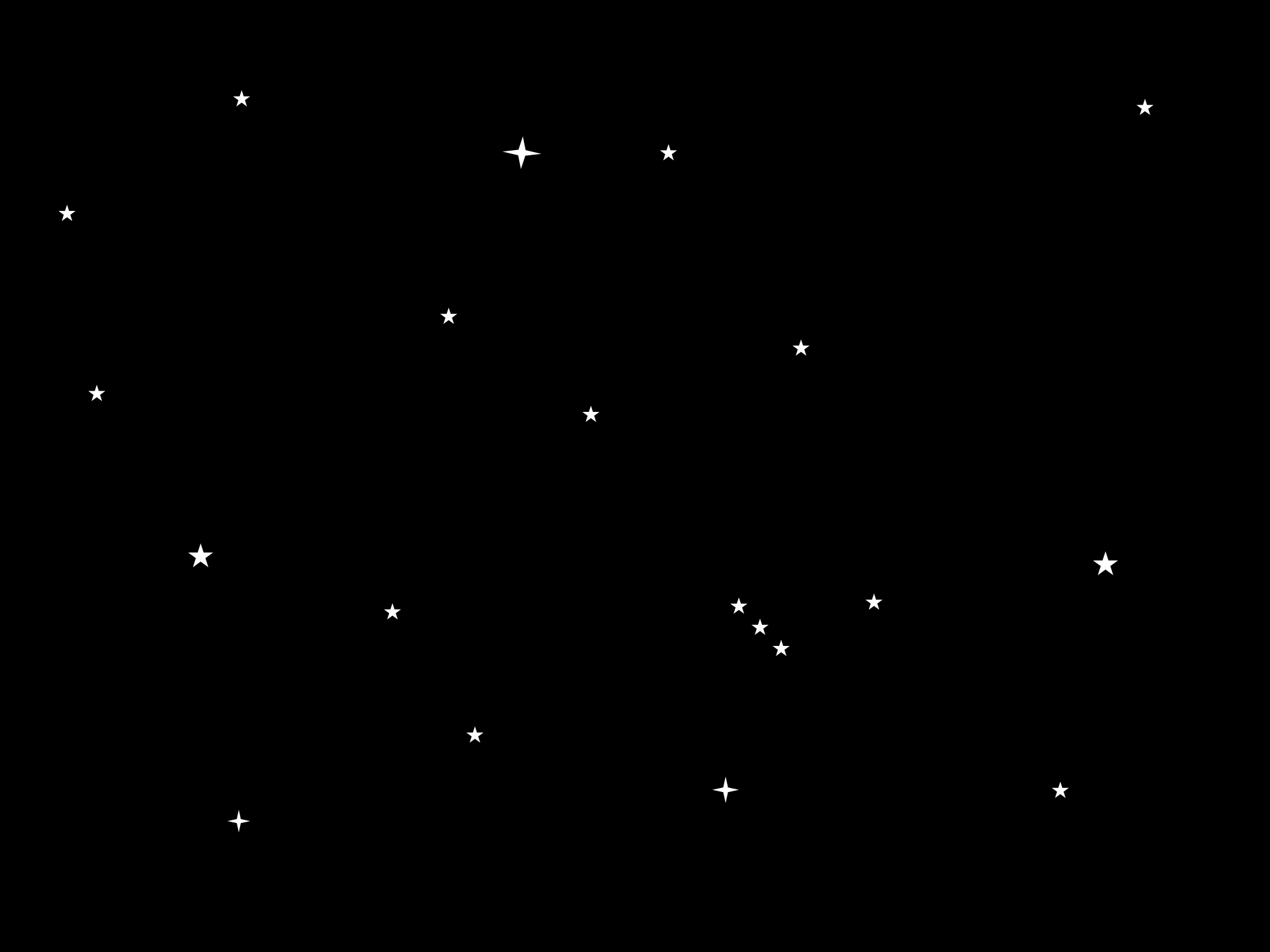


Little Belt Mts, Montana

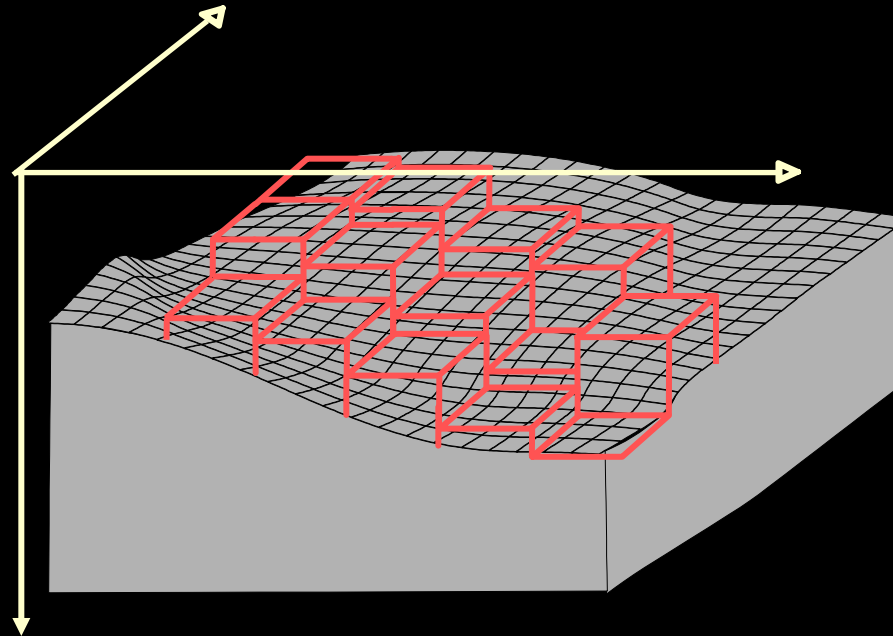


Little Belt Mts, Montana





# Mapeamento do relevo de uma interface

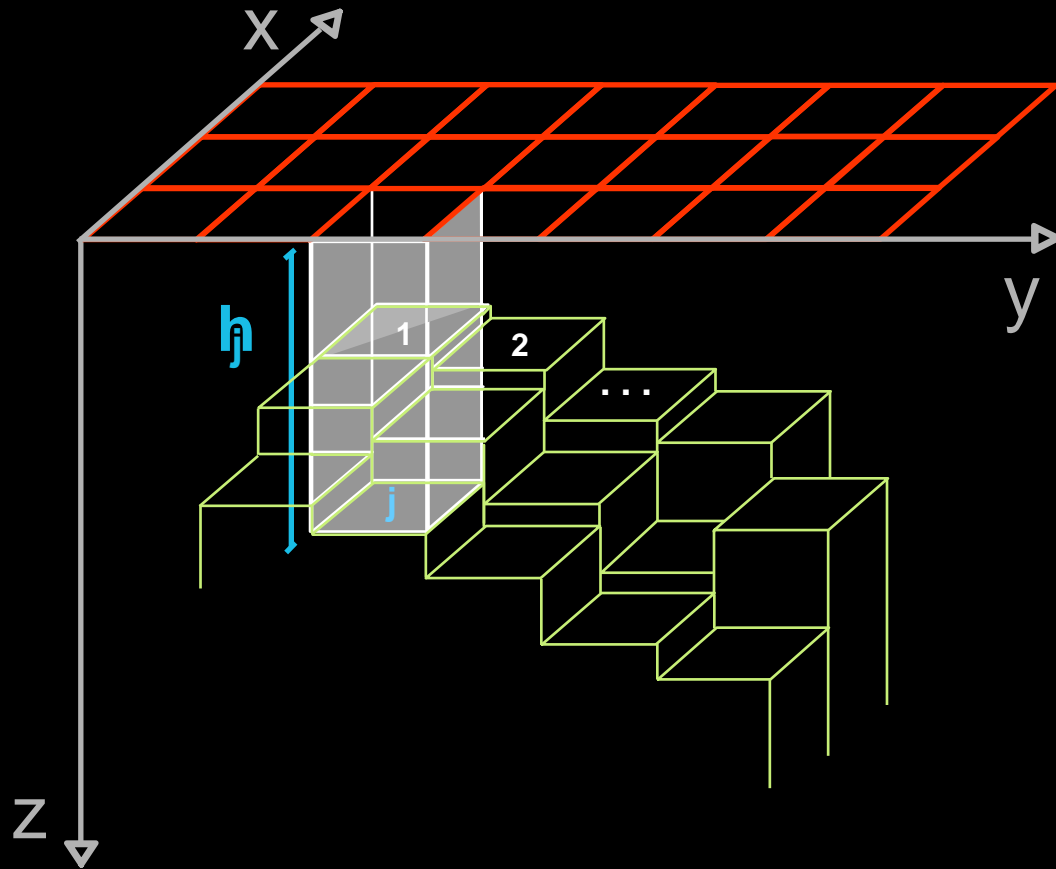


**Problema geológico:** mapeamento em planta do relevo de uma interface

**Simplificações:**

- a) a resposta geofísica é causada por uma única interface
- b) a interface separa dois meios homogêneos
- c) o contraste de propriedade física entre os dois meios é conhecido

# Modelo interpretativo



# Problema matemático:

$$f(p) = y^o$$

Para garantir existência de uma solução,  
minimiza-se:

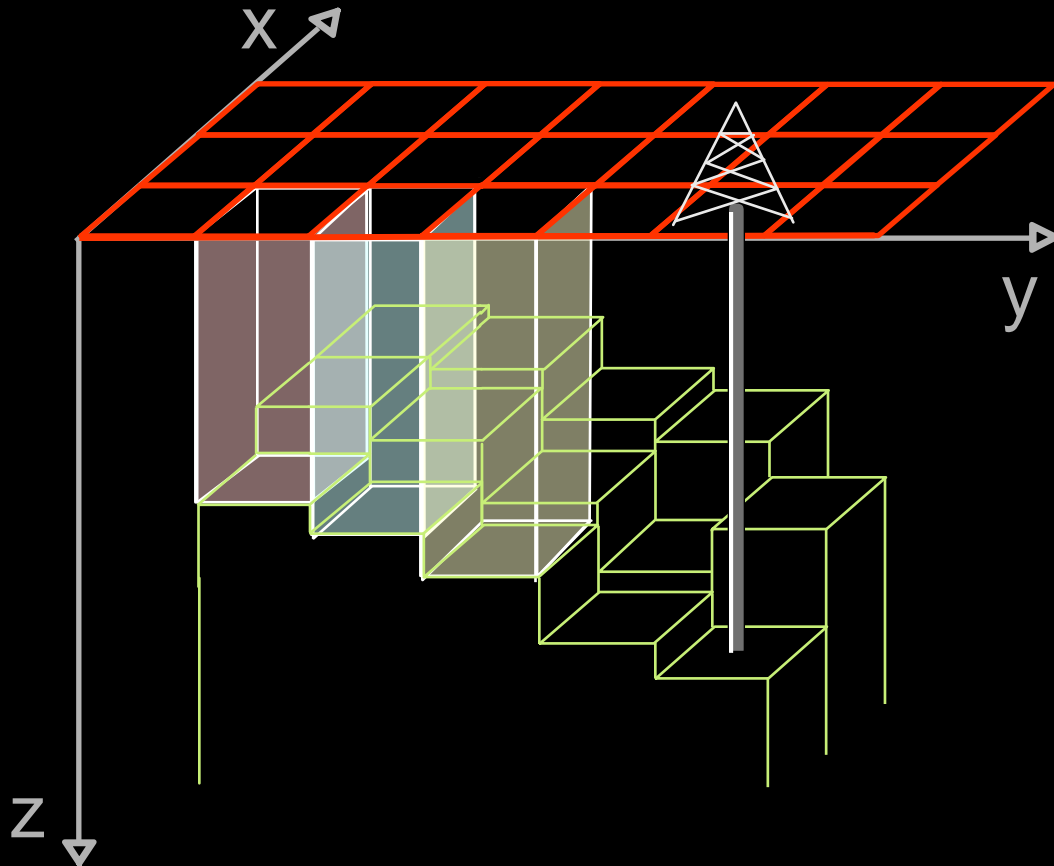
$$[y^o - f(p)]^T [y^o - f(p)]$$



# Estabilizadores:

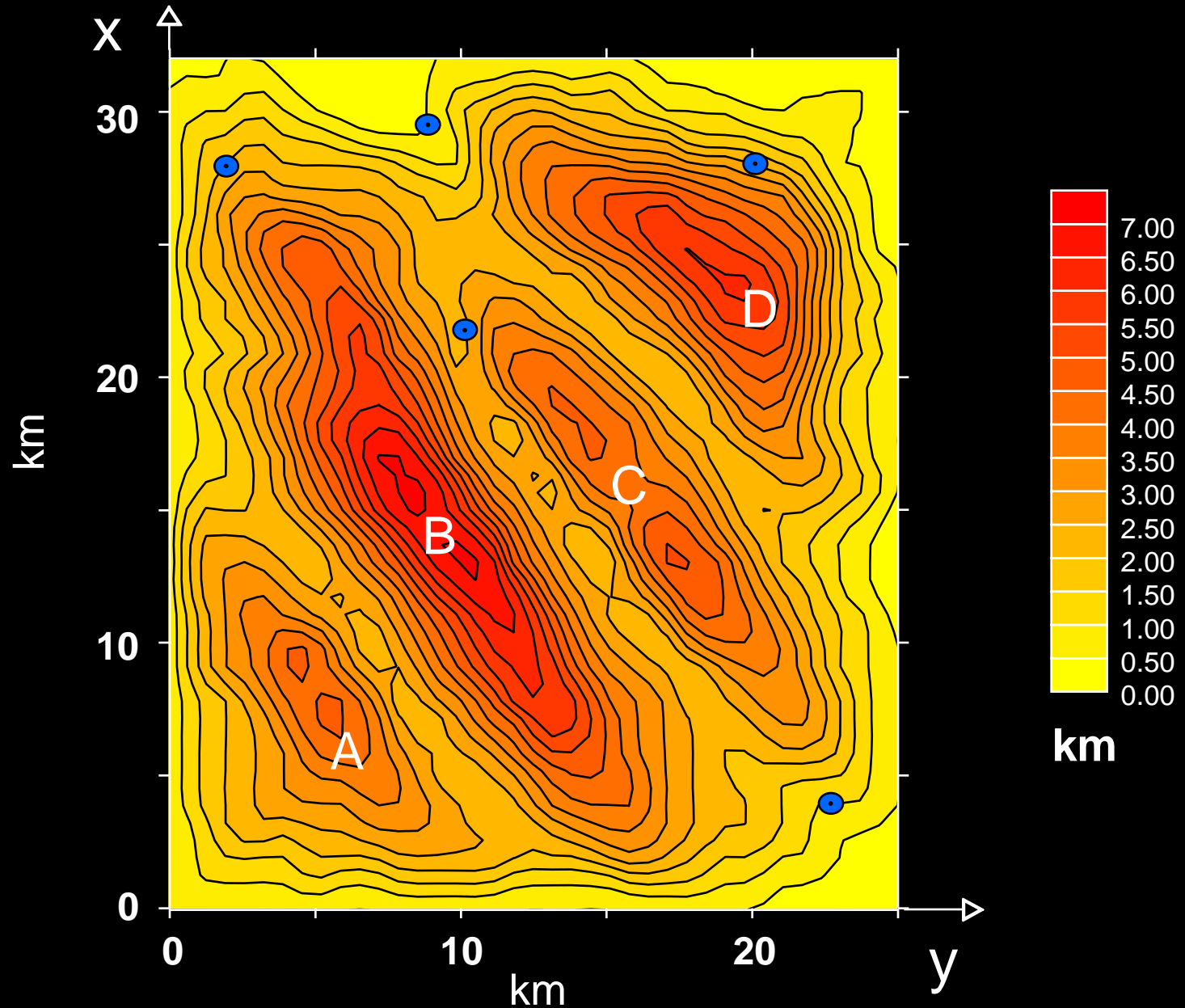
**Igualdade relativa:** proximidade entre todos os parâmetros adjacentes

**Igualdade absoluta:** proximidade de alguns parâmetros a valores conhecidos

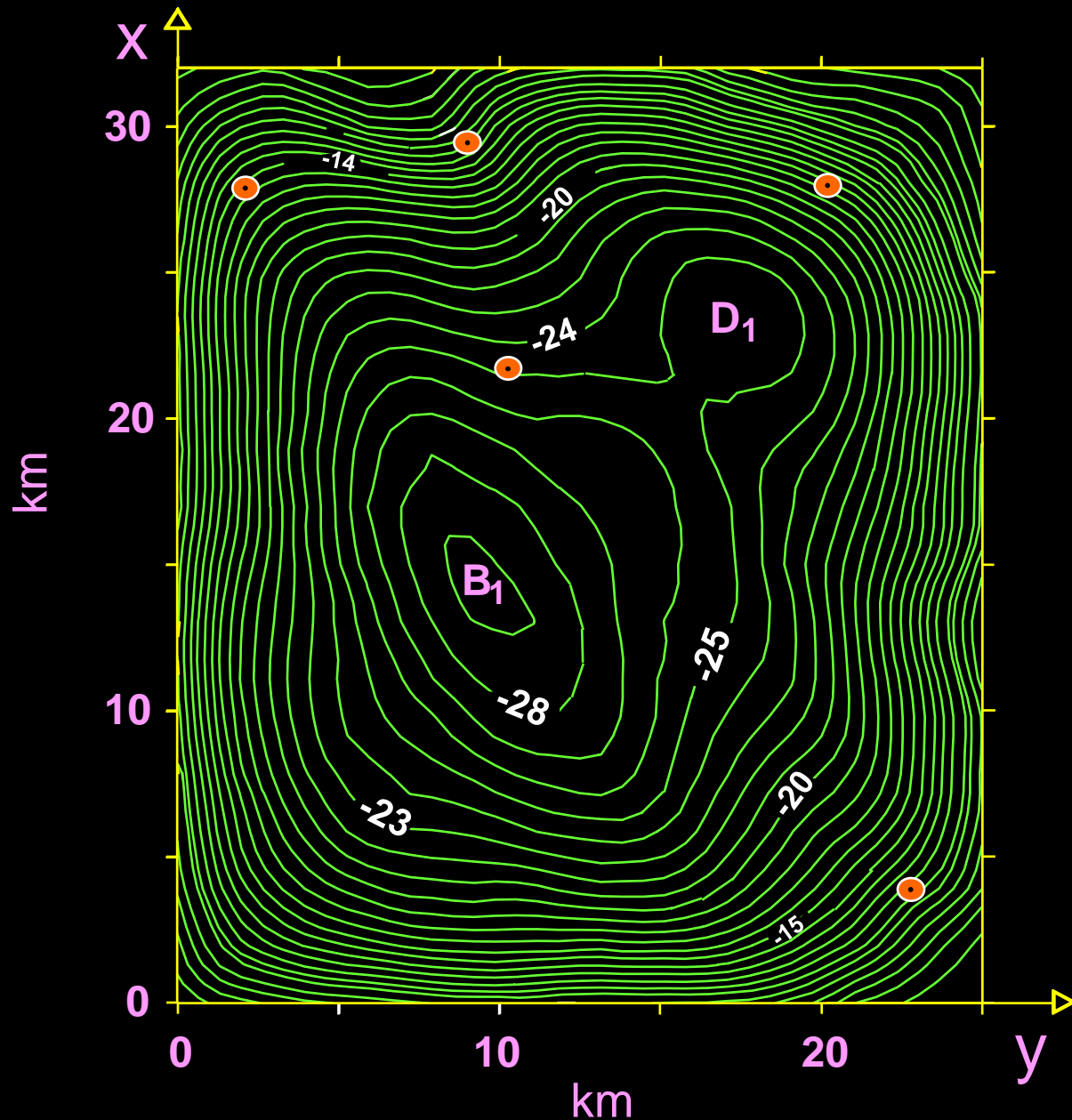


# Exemplo sintético

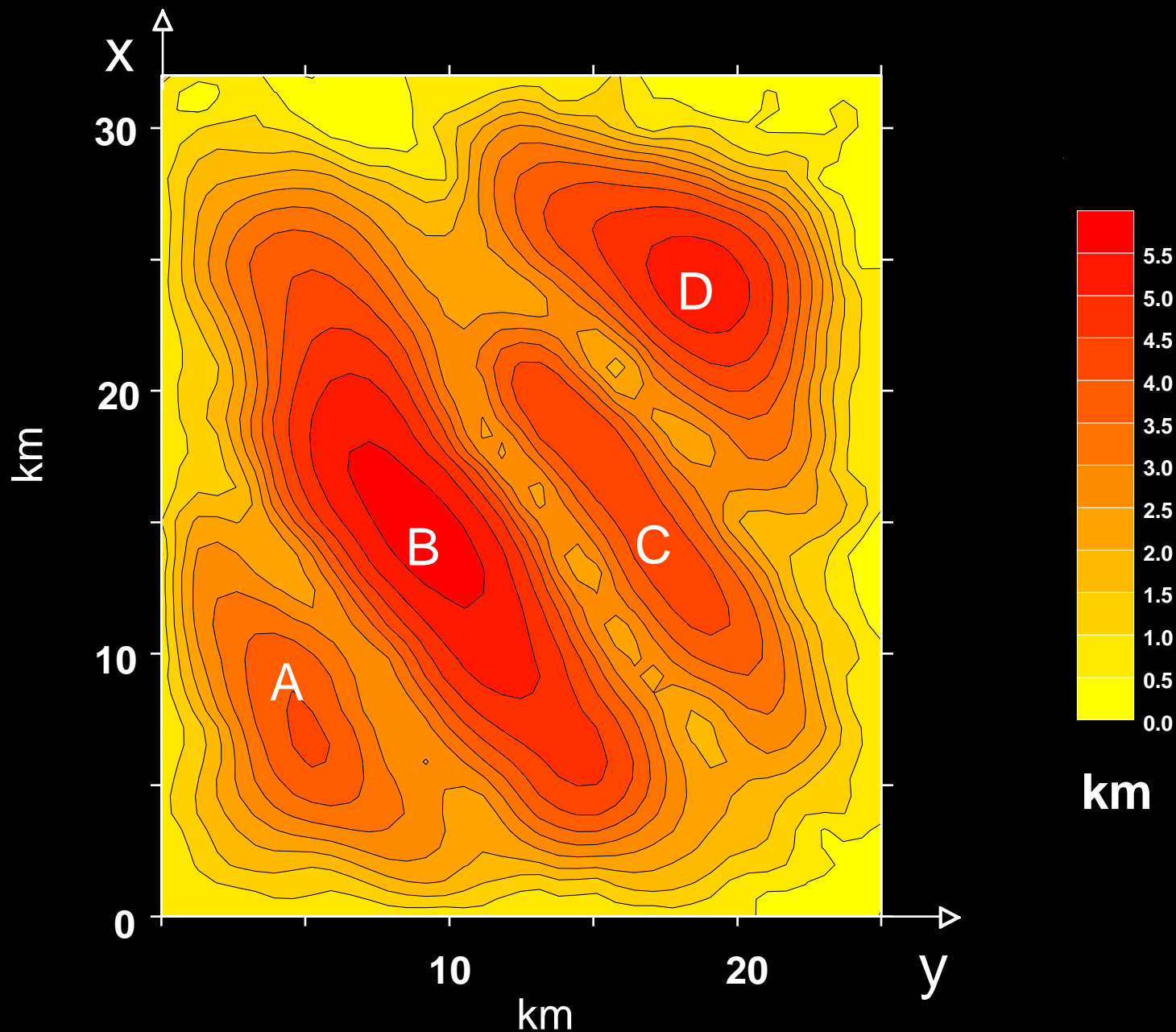
## Relevo verdadeiro



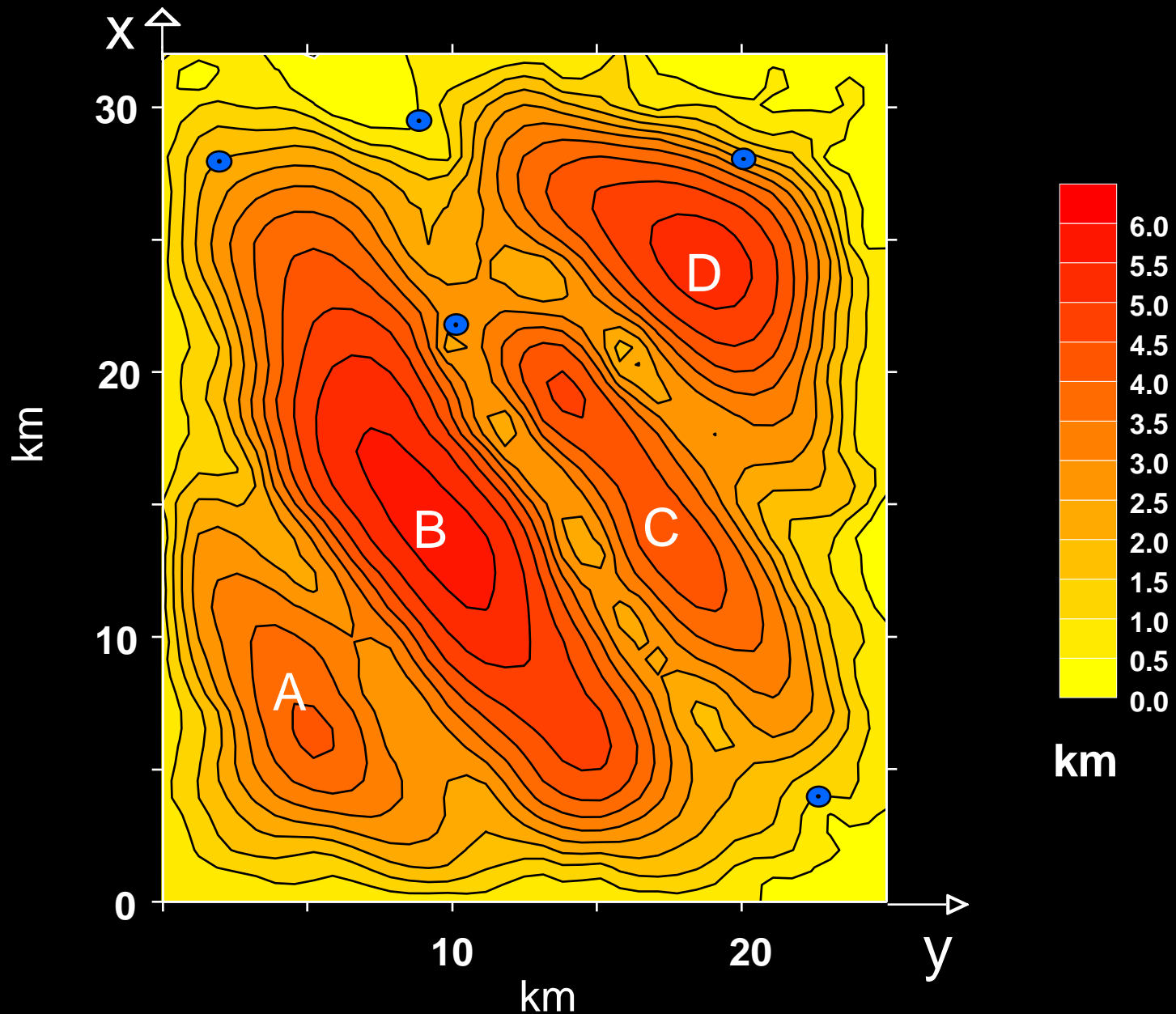
# Exemplo sintético: campo “observado”



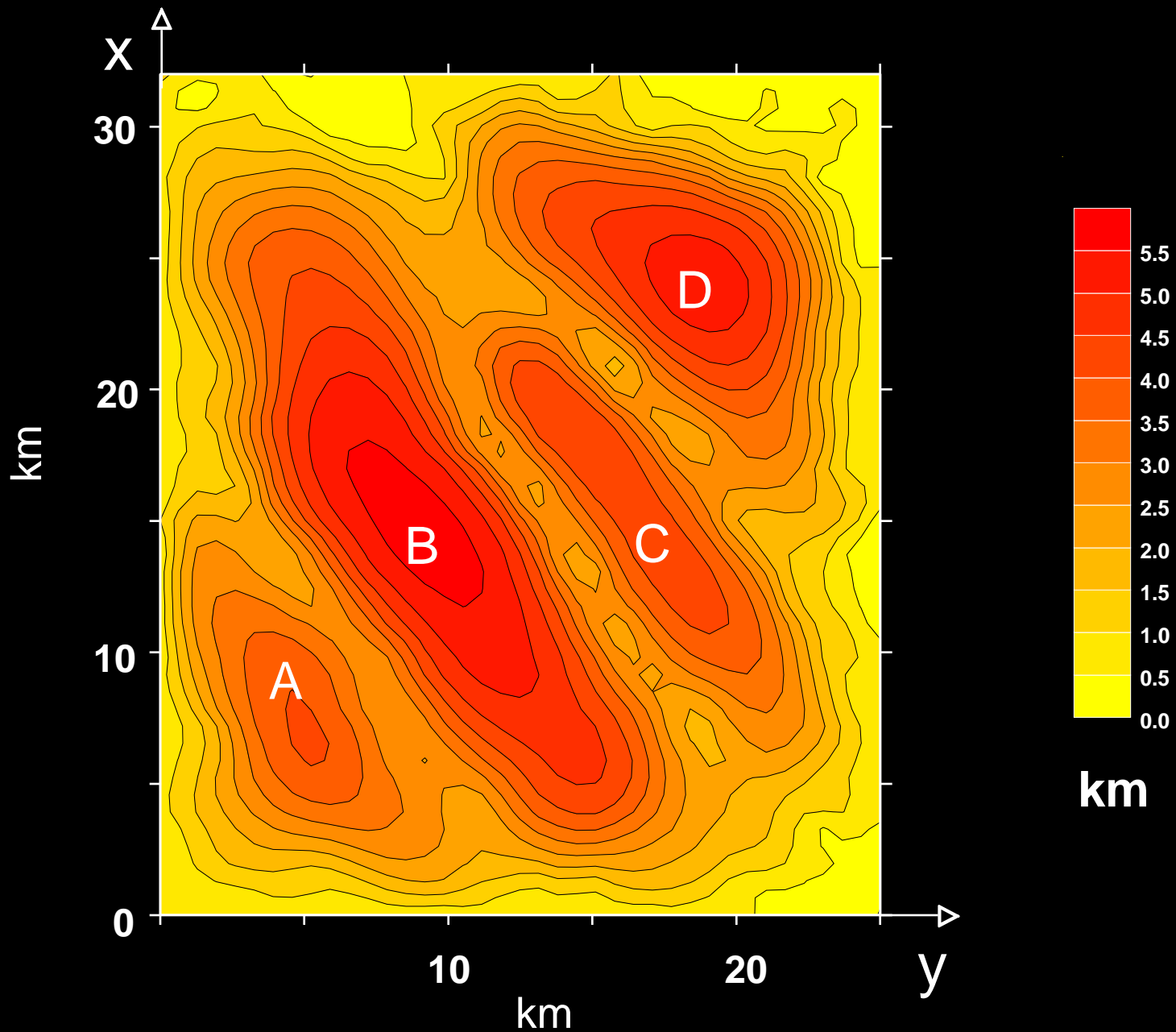
# Exemplo sintético: relevo estimado



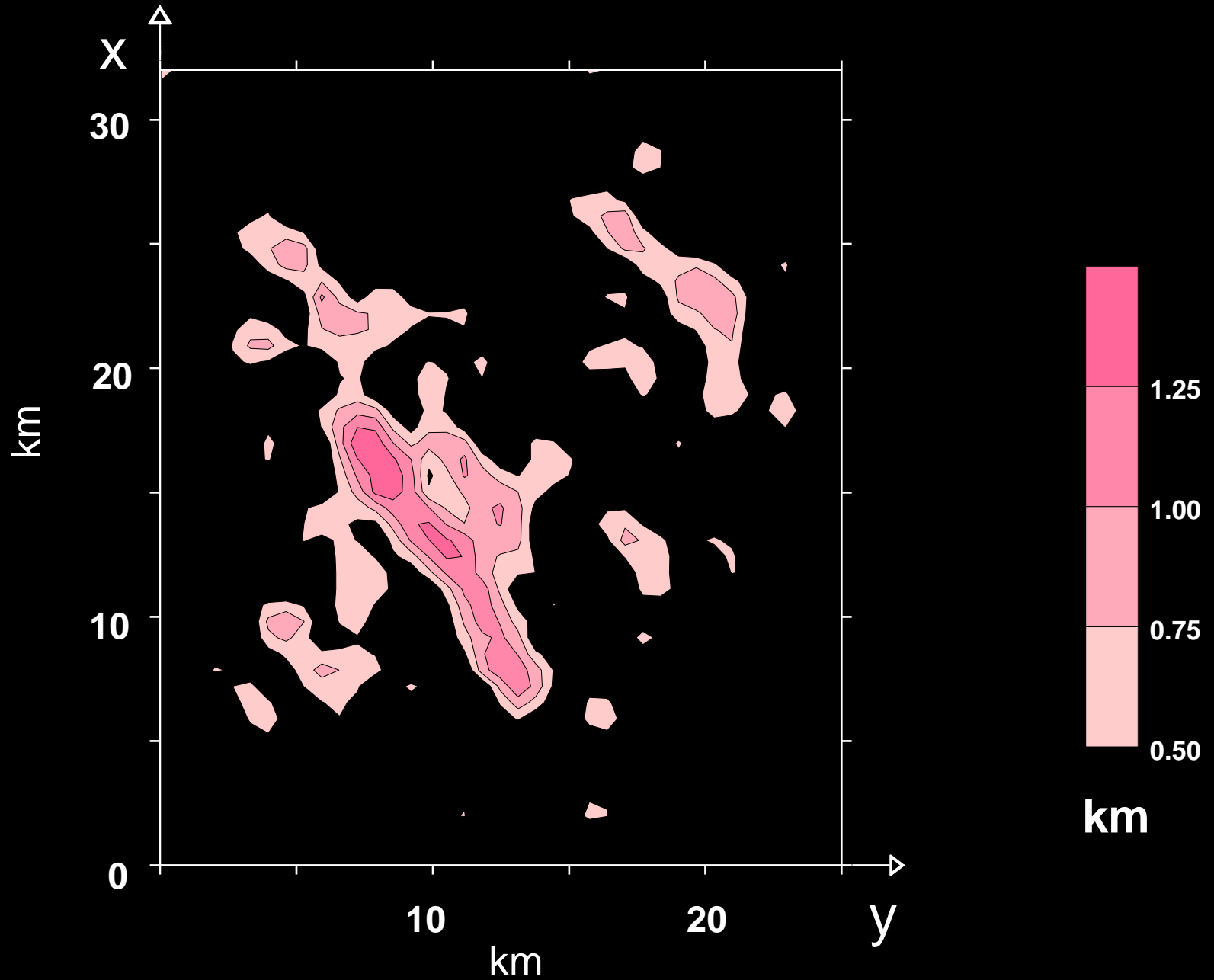
# Exemplo sintético: relevo estimado



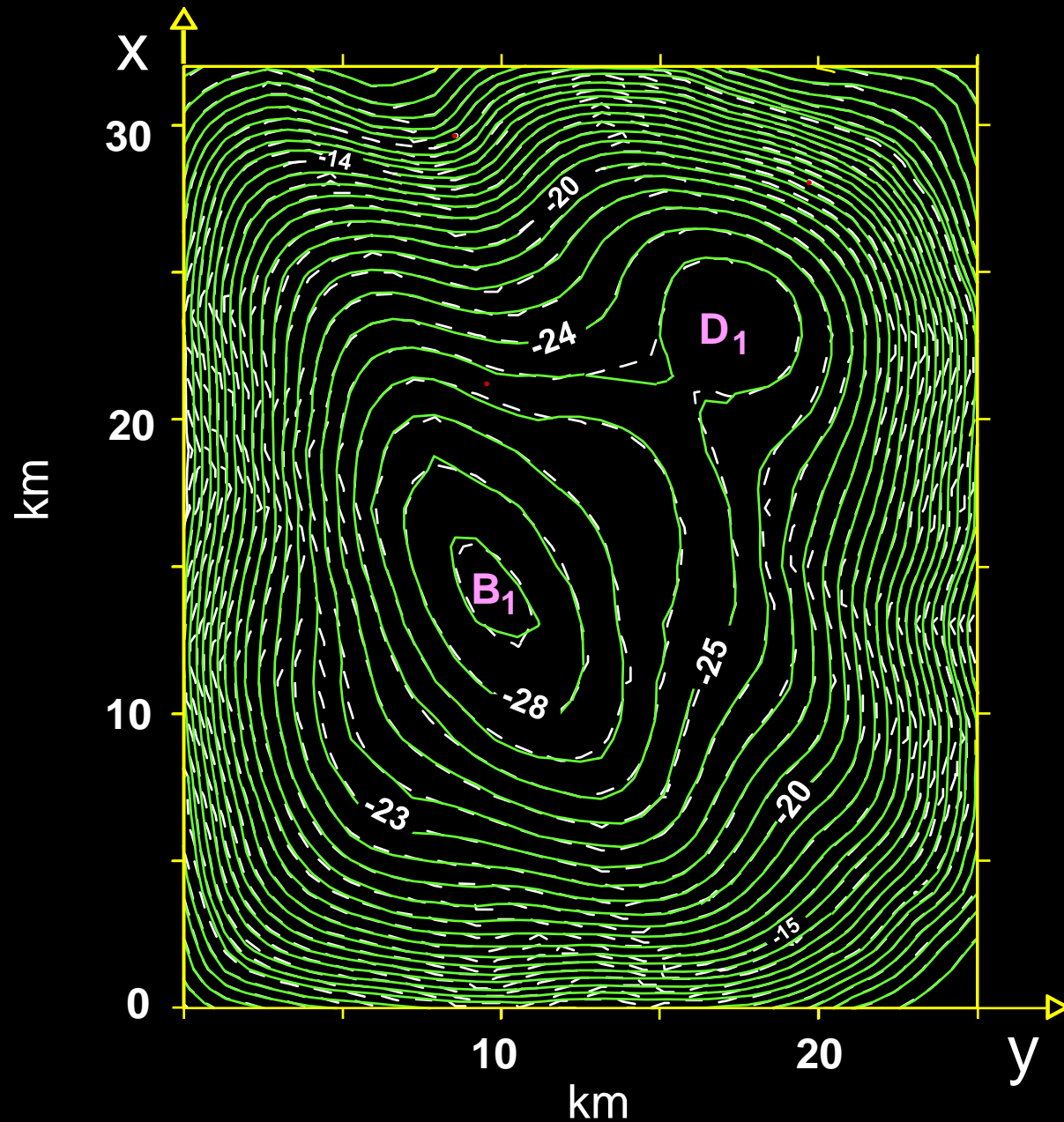
# Exemplo sintético: relevo estimado



## Exemplo sintético: relevo residual



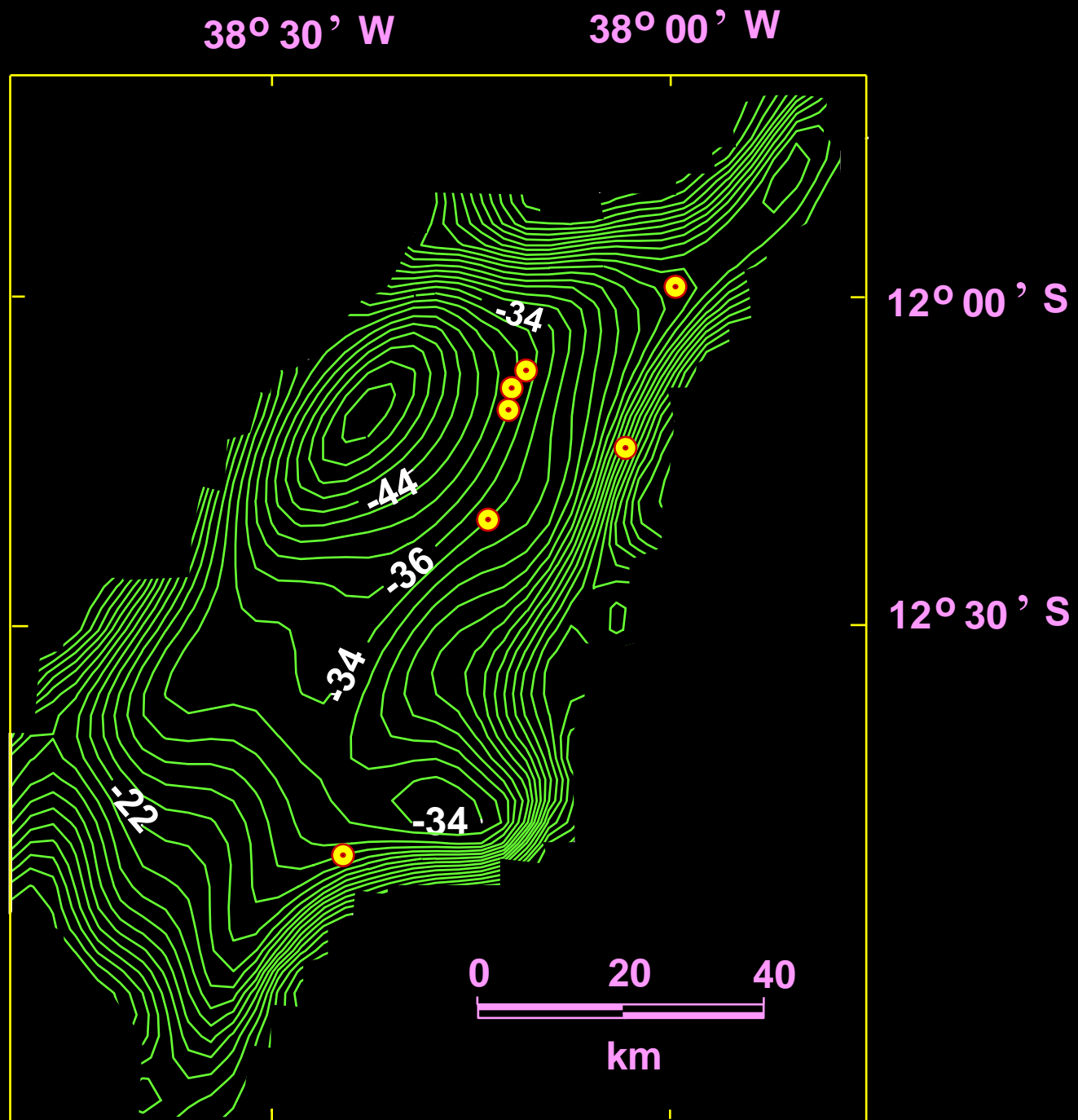
## Exemplo sintético: anomalias observada e calculada





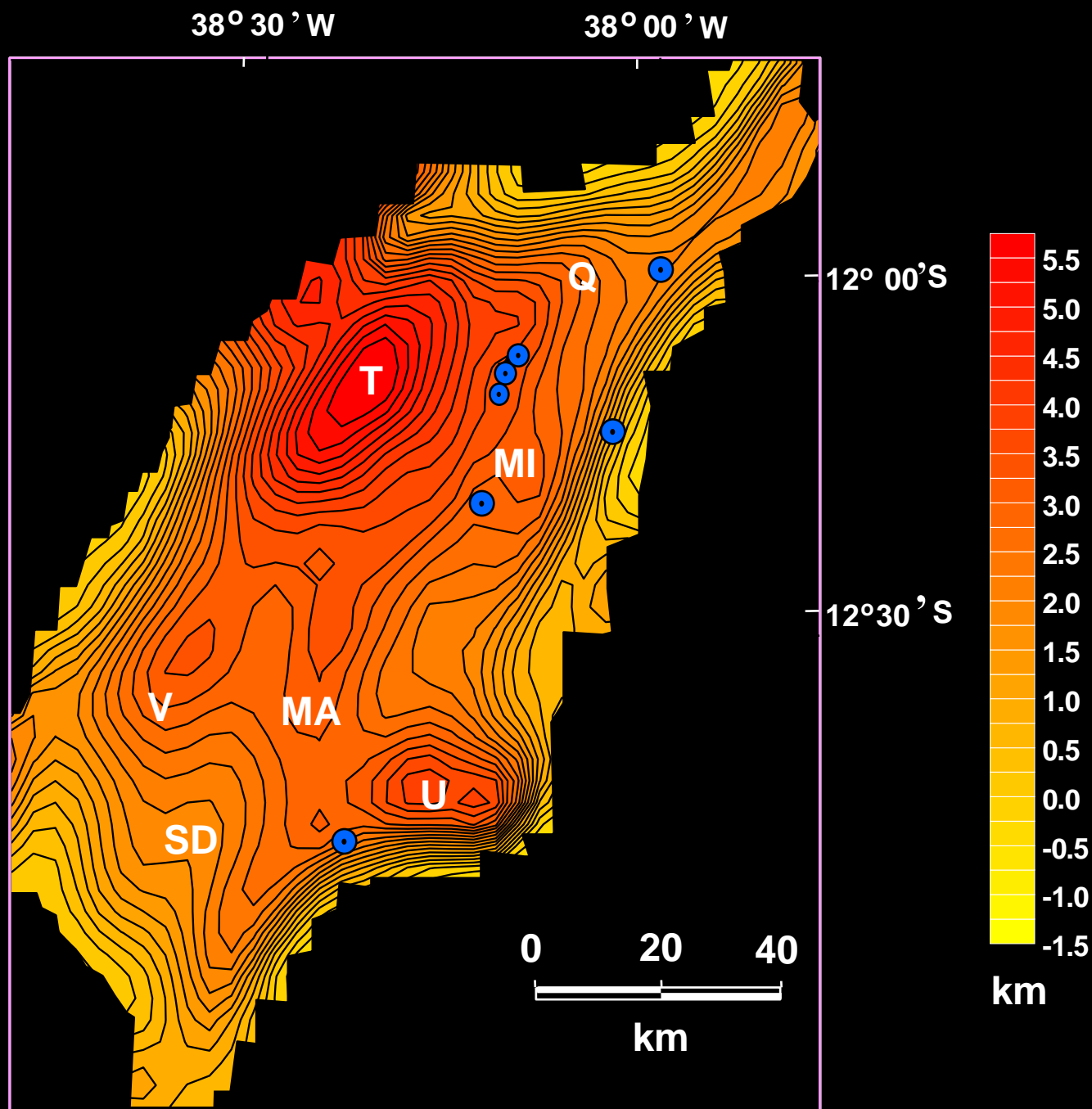
# Recôncavo

## Anomalia Bouguer



# Recôncavo

Relevo  
Estimado



# Recôncavo

Anomalias  
observada e  
calculada

